Georgia Mathematics Strategies Toolkit to Address Learner Variability for High School
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Introduction

“If the goal in mathematics teaching and learning is to support student success with mathematical proficiency, then we must be explicit about using instructional routines that focus on student engagement in activities that support reasoning and sense making, communication with and about mathematical ideas, making meaningful connections, building procedural fluency from conceptual understanding...”

- Thinking about Instructional Routines in Mathematics Teaching and Learning

Within this toolkit, educators will find observations of student behavior for each of the 14 identified areas for addressing learner variability. Aligned to each observation of student behavior, are evidence-based, research-based strategies intended to strengthen students’ ability in mathematics. Support resources are provided to assist educators with implementing the strategies.

- Behavior
- Cognitive Processing
  - Attention
  - Conceptual
  - Memory
  - Reasoning
- Executive Functioning
- Instructional Climate and Student Mindsets
- Language Processing
- Language Proficiency
- Mathematics Calculation
- Other Exceptionalities
- Problem-Solving
- Visual-Spatial Processing

Looking for more evidence-based, researched based practices for mathematics? Please visit gadoe.org/mathematics.
Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning. (EMTP 1)
2. Implement tasks that promote reasoning and problem solving. (EMTP 2)
3. Use and connect mathematical representations. (EMTP 3)
4. Facilitate meaningful mathematical discourse. (EMTP 4)
5. Pose purposeful questions. (EMTP 5)
6. Build procedural fluency from conceptual understanding. (EMTP 6)
7. Support efforts of learning in mathematics. (EMTP 7)
8. Elicit and use evidence of student thinking. (EMTP 8)

Mathematical Practices

1. Make sense of problems and persevere in solving them. (MP.1)
2. Reason abstractly and quantitatively. (MP.2)
3. Construct viable arguments and critique the reasoning of others. (MP.3)
4. Model with mathematics. (MP.4)
5. Use appropriate tools strategically. (MP.5)
6. Attend to precision. (MP.6)
7. Look for and make use of structure. (MP.7)
8. Look for and express regularity in repeated reasoning. (MP.8)
Behavior

Mathematics Connection: When students develop productive mathematical behavior, they are confident in their ability to attain proficiency and outstanding performance on an assignment. They are not afraid to ask questions or seek assistance when needed, and they are confident and display limited frustration when grappling with challenging problems.

Student Learning Expectations: Students should have opportunities to develop a growth mindset and transform to show a mathematical mindset by gaining confidence, taking instructional risks, and asking questions for clarity on challenging activities to achieve at their optimal performance.

<table>
<thead>
<tr>
<th>Behavior</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
<tbody>
<tr>
<td><strong>Observations</strong></td>
<td><strong>The teacher will support the efforts in learning mathematics. (EMTP 7)</strong>&lt;br&gt;• The teacher will provide students with manageable tasks.&lt;br&gt;• The teacher will anticipate where students may need additional supports during a lesson and be prepared to support them productively through the efforts of learning.&lt;br&gt;The teacher will ask questions that go beyond gathering information but allows them to identify where students need support. (EMTP 5)</td>
<td><strong>The student will reason abstractly and quantitatively while working in chunks to complete the task. (MP.1 &amp; 2)</strong>&lt;br&gt;• The student will utilize mathematics comprehension strategies to solve problems.&lt;br&gt;• The student will start to use metacognitive strategies to identify areas where they need support.</td>
<td><strong>3 Reads Instructional Routine</strong>&lt;br&gt;Designed to develop students’ ability to make sense of problems by deconstructing the process of reading mathematical situations.</td>
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<tr>
<td>Observations reveal signs of feeling overwhelmed resulting in shuts down during instruction.</td>
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<tr>
<td><strong>Observations</strong></td>
<td><strong>The teacher will facilitate discourse among students by positioning them as authors of ideas, who explain and defend their approaches. (EMTP 4)</strong>&lt;br&gt;• The teacher will encourage students to embrace the need for additional effort, make mistakes, and take risks through discourse and messaging in their classroom.</td>
<td><strong>The student will persevere and put forth effort in solving contextual, mathematical problems. (MP.1)</strong>&lt;br&gt;The student will explain their thinking and critique the reasoning of others during mathematics discussions. (MP.3)</td>
<td><strong>5 Tips for Improving Students’ Success in Math</strong>&lt;br&gt;Teachers who coach teams in a mathematical modeling challenge share ideas for helping students develop more confidence.</td>
</tr>
</tbody>
</table>
Cognitive Processing: Attention, Conceptual, Memory, Reasoning

Mathematics Connection: Cognitive processes include developmental processes such as attention, memory, reasoning, and conceptual processes; when used by students every day as a support strategy, these strategies will allow students to develop profound connections of the concepts, be able to recognize relationships, and perform cognitive procedures with situational-based problems. Once students make the connection, they can transfer that new acquired thinking to problem solving.

Student Learning Expectations: Students should have opportunities to use cognitive process strategies as a learning support tool to bridge the prior ideas with the new learning concepts. Students are now fully, equipped with the ability to perform the procedures to arrive at a solution.

Cognitive Processing: Attention

Mathematics Connection: Increasing complexity of mathematics content and tasks require extended attention spans. Maintaining attention can be challenging if a student is uninterested or the task seems too challenging.

Student Learning Expectations: Students should have opportunities to focus on a portion of any given information to concentrate on the relevant content, while using cognitive attention processes to remain focused on the pertinent information.

<table>
<thead>
<tr>
<th>Attention</th>
<th>Observations</th>
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<th>Student Actions</th>
<th>Support Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observations</strong></td>
<td>Observations reveal a lack of attentiveness on assignments. For example, common errors and mistakes when solving problems involving expressions and equations.</td>
<td>The teacher will praise students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. (EMTP 7) • The teacher will facilitate and establish a safe and positive learning environment for students to realize errors, mistakes and misconceptions.</td>
<td>The student will monitor and evaluate their progress and change course if necessary. (MP 1) The student will examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. (MP 1)</td>
<td>Mathematical Mindset Teaching Guide: User Advice Our goal for the guide is to support a mathematical mindset journey of learning and growth. Mathematical Mindset Practice 1: Growth Mindset Culture Praising the Learning Process</td>
</tr>
<tr>
<td><strong>Observations reveal limited focus when completing challenging tasks.</strong></td>
<td>The teacher will carefully select tasks to motivate students through opportunities for exploring and solving problems that extend their current mathematical understanding. (EMTP 2) • The teacher will create a cognitively demanding environment to build stamina as students persevere.</td>
<td>The student will persevere in exploring and reasoning through tasks. (MP 1) • The student will present their thinking and sustain stamina to preserve the difficulties and challenges. The student will use concrete, digital or computer tools to organize the math</td>
<td>Explicit Classroom Norms to Teach Kids How to Learn From Solving Problems This blog post is the fourth in a series of four blog posts exploring the student experience of problem-based learning. The first three posts are available here: (1) ”How Do Students Perceive Problem-Based Learning?” (2) ”Inviting Students to the...”</td>
<td></td>
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</table>
explore meaningful tasks. The teacher will provide purposeful questions to assess and gather evidence of students thinking while using visual representation to illustrate the skills and concepts during an evidence based close reading strategy. (EMTP 5) content to focus and illustrate their understanding during the close reading. (MP 5) mathematical modeling prompts and this guidance for how to use them represent our effort to make authentic modeling accessible to all teachers and students. 5 Strategies of developing conceptual understanding This is a free website providing teachers with strategies to develop students’ conceptual understanding of concrete concepts. Mathematical Modeling Prompts This website has a list of tasks designed with supporting conceptual learning using visual and numerical representations. Visual Representations Resources Visual Representations with Reasoning and Making Sense Tasks

<table>
<thead>
<tr>
<th>Cognitive Processing: Conceptual</th>
<th>Mathematics Connection: Conceptual understanding involves meaningfully learned and well-integrated knowledge about mathematics. Students should gain many logical connections between specific concepts and ideas.</th>
<th>Student Learning Expectations: Students should have opportunities to use conceptual processing to help make sense of contextual, mathematical problems to gain a deeper understanding of the integrated knowledge and the connections of the big idea concepts.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conceptual</strong></td>
<td><strong>Teacher Actions</strong></td>
<td><strong>Student Actions</strong></td>
</tr>
<tr>
<td>Observations reveal limited ways to solve non-routine, non-rote, and inquiry-based problems.</td>
<td>The teacher will select tasks that provide multiple entry points using varied tools and representations. (EMTP 2) The teacher will anticipate where students might need additional supports when solving non-routine problems and be prepared to support them productively through the efforts of learning. (EMTP 7)</td>
<td>The student will look closely at the structure and pattern of the content knowledge and reasoning skills based on the visual representations and models. (MP.7) The student will ask questions that are related to the task and will make progress in understanding and solving tasks. (MP.1, 2 &amp; 3)</td>
</tr>
<tr>
<td>Observations reveal errors with manipulating expressions, functions, and graphs.</td>
<td>The teacher will provide opportunities for students to use visual representations to support creating, drawing and constructing mathematics to make a deep understanding of the concepts. (EMTP 3)</td>
<td>The student will display making connections by interacting and engaging with visuals, concrete manipulatives, modeled representations when exploring and investigating questions. (MP.4) The student will demonstrate flexibility to</td>
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</table>
Cognitive Processing: Memory

Mathematics Connection: Using rote memory to recall facts or remember the steps of an algorithm does not yield long-term learning. Students should have opportunities to use conceptual learning strategies that will lead to committing the basic computational facts to memory.

Student Learning Expectations: Students should have opportunities to use knowledge from their memories to perform calculations and procedures, identify geometric figures, and demonstrate basic graphing skills by using visual-spatial and numerical representations to make sense of contextual, mathematical problems to help with sustaining long-term memory.

<table>
<thead>
<tr>
<th>Memory</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
<tbody>
<tr>
<td>Observations reveal limited recall of instructions and information from class discussions or assessments review.</td>
<td>The teacher will discuss and refer to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. (EMTP 1)</td>
<td>The student will use learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. (MP.1, 2, &amp; 6)</td>
<td>HLP 14: Teach Cognitive and Metacognitive strategies to Support Learning and Independence</td>
</tr>
<tr>
<td>Observations reveal inability to make connections between vocabulary, basic theorems and procedures when solving problems.</td>
<td>The teacher will provide the opportunities to use the visual-spatial representation to support the connection between basic theorems and vocabulary. (EMTP 3)</td>
<td>The student will identify relationships using visual tools to reason about numbers, vocabulary, expressions in different ways, such as words, symbols, pictures. (MP.1, 5, &amp; 7)</td>
<td>Math is Visual: Making Connections</td>
</tr>
</tbody>
</table>

This website has a list of tasks designed with supporting conceptual learning using visual and numerical representations.
# Cognitive Processing: Reasoning

**Mathematics Connection:** Mathematical reasoning enables students to make use of the concepts and skills in the lesson by gaining access to understanding. Students evaluate the situations, select problem-solving strategies, draw conclusions, develop and describe solutions and situational context. When students apply mathematical reasoning, they can reflect on the contextual, mathematical problems.

**Student Learning Expectations:** Students should have opportunities to critique and explain their thinking to make sense of the problem and the reasoning of others as they engage in contextual, mathematical situations.

<table>
<thead>
<tr>
<th>Reasoning</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</table>
| Observations reveal misunderstanding when developing logical conclusions or justifying an explanation from a calculated solution. | The teacher will ask open-ended questions for students to reason and make sense of evidence or solution. (EMTP 5)  
- The teacher will make certain questions go beyond gathering information to requiring explanation and justification. | The student will make conclusions and use equations, tables, graphs, diagrams, digital tools to communicate conclusions. (MP.1, 4 & 6)  
- The student will identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. | Learning Strategies for Reason and Making Sense Part 1  
A NCTM resource providing teachers strategies and methods to ensure reasoning is paramount in the classroom.  
[5 Ways to Improve Mathematical Reasoning](#)  
A free website providing teachers with strategies to improve mathematical reasoning skills. |
| Observation reveals inability to make informed connections of a new concept with prior knowledge during problem solving. | The teacher will provide opportunities for students to represent, discuss and make connections among mathematical ideas in multiple forms. (EMTP 3)  
The teacher will establish clear goals that articulate the mathematics that students are learning and the explicit connections to previous learning. (EMTP 1) | The student will use different representations to make the connections between the new and old concepts. (MP.7)  
The student will connect their current work with the mathematics that they studied previously and seeing where the mathematics is going. (MP.7 & 8) | Learning Strategies for Reason and Making Sense Part 1  
A NCTM resource providing teachers strategies and methods to ensure reasoning is paramount in the classroom. |
Executive Functioning

**Mathematics Connection:** Executive functioning is a set of cognitive mental competencies that students use to organize information and put it into use while working on mathematical problems, visualize problems from multiple angles, come up with different ways to solve them, and control attention and behavior to manage emotions when encountering the efforts of learning in mathematics.

**Student Learning Expectations:** Students should have opportunities to make sense of problems and persevere in solving them while making use of structure and reasoning about the repeated use of structure. Students will also be expected to model with mathematics.

<table>
<thead>
<tr>
<th>Executive Functioning</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</table>
| Observations reveal misunderstanding with comparing and analyzing quantities and relationships. | The teacher will anticipate students’ difficulties with quantities by connecting different student’s responses to the key math ideas. (EMTP 4) | The student will monitor their own learning by looking for repeated reasoning that may occur in the problems to compare quantities. (MP.7) | Instructional Strategies to Support Math Difficulties
This is a website to provide teachers with strategies with students experiencing math difficulties. |
| Observations reveal inability to manipulate or transform problems into an equivalent form or different representations. | The teacher will create and design opportunities for students to increase fluency and reasoning skills by encouraging students to use flexibility strategies of operations, objects, solutions, graphs and charts when displaying multiple forms. (EMTP 3 & 6) | The student will demonstrate adaptive and fluency skills using multiple representations. (MP.2) | 6 General Ways for Students to Think Flexibly
This is a free website to provide teachers with different math approaches to solving problems. |
Instructional Climate and Student Mindsets

Mathematics Connection: A student's mindset and disposition can influence how they understand, connect, and recognize relationships in mathematics. A connection between mathematical mindsets and learning must be addressed if mathematics is going to be accessible to all students. Students’ mathematical understanding is developed over time; therefore, it is important to recognize how students’ understanding of their abilities and the instructional climate can affect mathematics conceptual learning.

Student Learning Expectations: Students should have opportunities to see themselves as numerate contributors to the world around them and beyond and think deeply about mathematics as they prepare to enter the future workforce with the critical thinking and reasoning skills necessary for success in both the local and global workforces.

### Observations Climate and Student Mindsets

<table>
<thead>
<tr>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
<tr>
<td>Observations reveal student has a negative mathematics disposition. Ex: “I can’t do math.”, “I’m not a math person.”</td>
<td>The teacher will motivate student learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. (EMTP 2) The teacher will identify what counts as evidence of student progress toward mathematics learning goals. (EMTP 8)</td>
<td>The students will form connections with daily aspects of their lives (and classmates’ daily lives) and mathematical learning through teacher’s mathematical topic examples. (MP.1) The student will think deeply about the mathematics using critical thinking and reasoning skills necessary for success. (MP.1, 2 &amp; 3)</td>
<td>5 Teaching Strategies to Build Math Confidence This website discusses 5 growth mindset strategies for student s who lack math confidence.</td>
</tr>
<tr>
<td>Observations reveal a sense of irrelevance of mathematical concepts discussed in class.</td>
<td>The teacher will implement tasks and provide opportunities that bridge the mathematics classroom with the community. (EMTP 2) • The teacher will view students as competent mathematical beings whose lived experiences are leveraged during mathematics instruction.</td>
<td>The student will construct confidence of who they are as mathematicians by using their norms and beliefs. (MP.1, 2, &amp; 3) The student will express self-love, pride, confidence and healthy self-esteem about themselves and their community as mathematical thinkers and learners. (MP. 3)</td>
<td>Resources to Increase Students Interest This is a website with different resources to increase student interest.</td>
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</table>
**Language Processing**

**Mathematics Connection:** Language processing is essential to mathematical discourse. Mathematical discourse is about the exchange of mathematical ideas, including ways of representing, thinking, talking, agreeing, and disagreeing. Language processing involves students’ ability to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing.

**Student Learning Expectations:** Students should have opportunities to express mathematical ideas with precision and clarity.

<table>
<thead>
<tr>
<th>Language Processing</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
<tr>
<td>Observations reveal a struggle with formulating questions.</td>
<td>The teacher will provide sufficient wait time so that students have time to formulate thoughts or questions. (EMTP 5)</td>
<td>The student will understand that they can ask useful questions to clarify or improve understanding. (MP.1)</td>
<td>Inside Mathematics/Standard 3 Construct Viable Arguments and Critique the Reasoning of Others. This website explains Standard 3 in detail at every grade level and provides video of teacher interactions with students.</td>
</tr>
<tr>
<td>Observations reveal an inability to explain, justify, or prove their reasoning for solutions.</td>
<td>The teacher will provide a learning environment that encourages thoughtful discussion. (EMTP 5)</td>
<td>The student will be able to analyze situations, justify conclusions, and communicate them to others. (MP.3)</td>
<td>Mathematical Language Routines Presentation from Illustrative Mathematics about various language routines.</td>
</tr>
</tbody>
</table>
Language Proficiency

**Mathematics Connection:** Language proficiency is essential to mathematical literacy and comprehension. Language proficiency involves students’ ability to understand and use the language of mathematics to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing.

**Student Learning Expectations:** Students should have opportunities to precisely use mathematical language to explain their thinking and reasoning in mathematics.

<table>
<thead>
<tr>
<th>Language Proficiency</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
<tr>
<td>Observations</td>
<td>Teacher Actions</td>
<td>Student Actions</td>
<td>Support Resources</td>
</tr>
</tbody>
</table>
| Observations reveal difficulty describing strategies, justifying solutions, or explaining their reasoning. | The teacher will choose tasks that engage students in meaningful mathematical discussions. (EMTP 2 & 5) | The student will be able to analyze situations, justify conclusions, and communicate them to others. (MP.1, 2 & 3) | Using the 5 Practices in Mathematics Instruction  
High-quality mathematical discourse involves choosing a task that’s worth talking about. |
| Observations reveal an inability to use the language of mathematics in writing to communicate mathematical ideas with precision and clarity. | The teacher will model the use of the language of mathematics in their writing as well as how to claim an answer and support it. (EMTP 4) | The student will justify their answers with the use of the language of mathematics in their writing. (MP.2 & 3) | Promoting Mathematical Argumentation  
This website provides examples and recommendations for teachers of how to promote mathematical argumentation. |
| Observations reveal an inability to use the language of mathematics orally to communicate mathematical ideas with precision and clarity. | The teacher will model the use of the language of mathematics in their oral response as well as how to claim an answer and support it. (EMTP 4 & 5)  
The teacher will model usage of sentence starters-frames to use the mathematical language. | The student will justify their answers with the use of the language of mathematics in their oral response. (MP.3) | Math Language Routines  
Illustrative Mathematics blog post on using math language routines with English Language Learners.  
Sentence Stems to Encourage Math Talks  
This website provides “25 Sentence Stems to Encourage Math Talk” along with a guide to how to use the sentence stems in a math class. |
Mathematics Calculation

**Mathematics Connection:** Mathematical calculation involves using mathematical or logical reasoning to solve a problem. When demonstrating math computation, students understand and use mathematical language, learn processes and procedures for solving math problems, and reflect on solutions to problems to determine if they make sense.

**Student Learning Expectations:** Students should have opportunities to show growth development in number sense to perform math calculations fluently by using the connections of understanding between visual-spatial and numerical representations.

<table>
<thead>
<tr>
<th>Mathematics Calculations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
<tr>
<td>Observations reveal repetitive calculations errors with algebraic equations and expressions.</td>
<td>The teacher will use visual models to support students’ understanding of general methods. (EMTP 6) The teacher will anticipate where students may need additional supports during the problem-solving process and be prepared to support them with strategic strategies. (EMTP 7)</td>
<td>The student will explain the mathematical basis for the procedures that they use and use procedures appropriately and efficiently. (MP.1, 2, 4, 6 &amp; 7) The student will ask questions that are related to the source of their struggles to make progress in understanding and solving tasks. (MP.1, 2 &amp; 7)</td>
<td>Secondary Number Talks This website provides various examples of number talk activities and how to implement these activities in the classroom.</td>
</tr>
<tr>
<td>Observations reveal difficulties maintaining precision when solving mathematical problems.</td>
<td>The teacher will provide immediate feedback by implementing an aggressively monitoring strategy during the learning phase where students are practicing the new acquired content. (EMTP 8)</td>
<td>The student will make sense of the concepts and relationships by demonstrating the ability to use old learning experiences. (MP.2)</td>
<td>GSE Effective Instructional Practices Guide This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices.</td>
</tr>
<tr>
<td>Observations reveal calculations error on mathematical contents when applying new content and knowledge.</td>
<td>The teacher will anticipate misconceptions and provide an opportunity to gather evidence of students’ misunderstandings on the new content. (EMTP 8)</td>
<td>The student will reflect on misconceptions and mistakes with creating logical progressions of constructed arguments from the procedural content. (MP.2 &amp; 3)</td>
<td>Guide for Effective Mathematics Instruction These mathematics toolkits provide strategies to identify gaps in learning and evidence-based strategies to address the gaps.</td>
</tr>
</tbody>
</table>
Other Exceptionalities

**Mathematics Connection:** Instruction must incorporate strategies and supports providing access to the content within the general education curriculum for mathematics, rich learning tasks that allow for the implementation of the Mathematical Practices and the building of proficiency in mathematics for all learners through sustained and deep engagement in practices that include problem solving, reasoning, and critiquing.

**Student Learning Expectations:** Students should have opportunities to meet high academic standards and demonstrate the level of mathematical reasoning needed to fully develop their conceptual understanding and procedural fluency.

<table>
<thead>
<tr>
<th>Other Exceptionalities</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
<tbody>
<tr>
<td>Observations reveal a struggle to demonstrate understanding through a singular mode of assessing.</td>
<td>The teacher will understand different learning styles and provide several modes of teaching and assessing student understanding. (EMTP 8)</td>
<td>The student will exhibit understanding through alternative modes of assessments. (MP.4 and 5)</td>
<td>How to Accommodate Different Learning Styles A UMass resource that provides teachers with characteristics and tips for accommodating different learning styles.</td>
</tr>
<tr>
<td>Observations reveal trouble accessing the curriculum during routine instruction due to proximity.</td>
<td>The teacher will provide environmental adaptations such as preferential seating.</td>
<td>The student will sit close to the teacher to enable him/her to enhance their learning.</td>
<td>Inclusive Teaching This website provides teaching and assessment strategies for students who are deaf/hard of hearing.</td>
</tr>
<tr>
<td>Observations reveal a struggle to see the whiteboard or instructional material.</td>
<td>The teacher will provide preferential seating to the student. The teacher will provide enhanced print on classroom assignments.</td>
<td>The student will access the information on the board or instructional materials to enable him/her to enhance their learning.</td>
<td>Adaptations for student with Visual Impairments This website provides information on mobility adaptations for all learners.</td>
</tr>
</tbody>
</table>
# Problem-Solving

**Mathematics Connection:** Problem-solving is a cognitive process that involves students learning mathematics through contextual, mathematical problems and models. The contexts and models allow students to build meaning for the concepts so they can fully understand abstract concepts.

**Student Learning Expectations:** Students should have opportunities to use mathematical reasoning to make sense of the content, extract information and explore new concepts as they persevere in solving contextual, mathematical problems.

## Problem Solving

<table>
<thead>
<tr>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</table>
| Observations reveal a struggle with developing a method or plan to reach a desired solution. | The teacher will provide multiple strategies to promote reasoning and problem solving so students develop cognitive strategies to arrive at a solution. (EMTP 3 & 4) | The student will identify one or multiple ways to approach a problem and devise a plan to reach a solution. (MP.1, 2, 4, & 5)  
- The student will identify models that are most efficient for solving specific problems or representing specific math ideas. | GSE Effective Instructional Practices Guide  
This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices. |
| Observations reveal lack of comprehension with context in a word problem to develop a visual, numerical, or model representation. | The teacher will use various cognitive strategies to support students’ ability to transform a word problem from its current state into a different representation to arrive at a solution. (EMTP 4 & 7) | The student will communicate the meaning of contextual problems and use precise language. (MP.3 & 6)  
The student will organize the information and manipulate the information to create visual or model representation. (MP.1, 4, & 5) | 3-Read Protocol  
The Three Read Protocol is designed to engage students in sense-making of language-rich math problems or tasks.  
Reading in the Mathematics Classroom  
Literacy Strategies for Improving Mathematics Instruction |
| Observations reveal lack of problem-solving skills. | The teacher will use various cognitive strategies to support students’ ability to transform a word problem from its current state into a different representation to arrive at a solution. (EMTP 4 & 7) | The student will construct meaning to the information necessary in the math process. (MP.1) | 9 Strategies to Improving Problem-Solving  
This website gives great insight to different techniques for effective questioning in the classroom. |
Visual-Spatial Processing

Mathematics Connection: Students create and use visual representations to solve problems and to explore and communicate mathematical concepts and ideas in the mathematical domains. Visual-spatial processing affects students’ ability to solve multi-step problems, analyze certain representations, interpret and manipulate geometric configurations and appreciate changes in objects as they are moved in space.

Student Learning Expectations: Students should have opportunities to experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, and creating equations. Students will be expected to connect the different representations and explain the connections.

<table>
<thead>
<tr>
<th>Visual-Spatial Processing</th>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
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<tr>
<td>Observations reveal a</td>
<td></td>
<td>The teacher will increase the processing time for students to visually represent the information obtained. (EMTP 4 &amp; 7)</td>
<td>The student will articulate the strategies they use to solve problems. (MP.1)</td>
<td>Concrete Representational Abstract A three-step instructional approach that has been found to be highly effective in teaching math concepts.</td>
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<tr>
<td>struggle with organizing</td>
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<td>information and creating</td>
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<td>a meaningful pattern.</td>
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</table>

| Observations reveal       | The teacher will provide visual representation by using math manipulatives, by letting students create sketches or graphs to show their mathematical thinking and by demonstrating new mathematical concepts through highly visual-spatial demonstrations. (EMTP 3) | The student will use multiple forms of representations such as visual media or diagrammed instruction to understand relationships between objects. (MP.5) | 11 ways to teach visual-spatial learners Techniques to help spatially intelligent students use their visual thinking skills in mastering mathematics |
| difficulty comprehending   | The teacher will increase the processing time for students to visually represent the information obtained. (EMTP 4 & 7) | The student will articulate the strategies they use to solve problems. (MP.1) | Concrete Representational Abstract A three-step instructional approach that has been found to be highly effective in teaching math concepts. |
| spatial relationships      | The teacher will provide visual representation by using math manipulatives, by letting students create sketches or graphs to show their mathematical thinking and by demonstrating new mathematical concepts through highly visual-spatial demonstrations. (EMTP 3) | The student will use multiple forms of representations such as visual media or diagrammed instruction to understand relationships between objects. (MP.5) | 11 ways to teach visual-spatial learners Techniques to help spatially intelligent students use their visual thinking skills in mastering mathematics |
| between 2-D and 3-D objects. | The teacher will increase the processing time for students to visually represent the information obtained. (EMTP 4 & 7) | The student will articulate the strategies they use to solve problems. (MP.1) | Concrete Representational Abstract A three-step instructional approach that has been found to be highly effective in teaching math concepts. |
Descriptions

**Behavior**
Behavior is a form of communication. Every behavior has a function. A growth mindset in mathematics can be demonstrated through the students’ confidence and perseverance in their thinking and executing, asking questions for clarity, and grappling with context-based problems. These students are willing to tackle challenging problems and take learning risks.

**Cognitive Processing**
Cognitive processes allow students to make connections between prior concepts and new ideas. When students are aware and use at least one of the four mathematics cognitive processes, they will be capable of transferring cognitive skills into their learning.

**Attention**
Attention is a cognitive process that involves the student's ability to listen to directions and explanations, participate in class discussions, focusing on different parts of the activity and stimuli at various moments in the lesson, and the ability to sustain an undisclosed amount of time dedicated to working and thinking on an activity for any given time range. Paying attention is learned.

**Conceptual**
Conceptual learning is when students use the previous learning experiences, ideas and concepts to understand and develop the new meaning of the new content by recognizing the connection and relationship of the old and new “building block concepts.”

**Memory**
Both long-term and short-term memory play essential roles in the learning of mathematics. Math is a conceptual subject consisting of connections and relationships.

**Reasoning**
Reasoning is the capacity to reflect on work, evaluate it, and then adapt, as needed.

**Executive Functioning**
Executive functioning is closely aligned to behavior and is judged by the strength of:
- self-awareness (self-directed attention),
- inhibition (self-restraint),
- non-verbal working memory (the ability to hold things in your mind),
- verbal working memory (self-speech),
- emotional self-regulation (learning to use words, images and your own self-awareness to process and alter how you feel about things),
- self-motivation (how well you motivate yourself to complete a task) and
- planning and problem solving (how you organize information in your brain).

**Instructional Climate and Student Mindsets**
Positive instructional climate and student mindsets can be developed through experiences obtained from physical location, ideas, and expectations of those within the environment. Classroom opportunities that demonstrate an awareness of and embrace all students’ ability to achieve success in mathematics can increase student autonomy and academic outcomes. Student autonomy encourages students to speak from their mathematical perspective and advocate for their own reasoning. Positive school climate leads to encouraging and supportive classroom environments that ultimately lead to growth mindsets and increased agency for each and every student.
**Language Processing**
Language processing involves students’ ability to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing. Language processing refers to the way humans use words to communicate ideas and understandings, and how such communications are processed and understood.

**Language Proficiency**
Mathematics language proficiency is measured in terms of interpretive and expressive language skills at the word/phrase/representation, sentence and discourse dimensions.

**Mathematics Calculation**
Mathematical calculations are basic computational skills (with and without the appropriate grade level calculator) and basic graphing skills that support the mathematical language, when solving contextual, mathematics problems. As a result of focusing on conceptual understanding, over time students develop procedural fluency (accurate, efficient, and flexible) and learn to make sense of and solve problems using automaticity of facts rather than rote recall.

**Other Exceptionalities**
Students with other exceptionalities may experience physical, social or emotional barriers that have the potential to influence how they engage in the learning experiences. Instructional opportunities should be limitless for all students to assist them with meeting high academic standards and developing mathematical reasoning and conceptual understanding of the content expectations.

**Problem-Solving**
Problem-solving is a cognitive process that involves students learning mathematics through contextual, mathematical problems and models. The contexts and models allow students to build meaning for the concepts so they can fully understand abstract concepts. Students are expected to use problem-solving strategies and develop these skills over time as they progress in the K-12 curriculum.

**Visual-Spatial Processing**
Visual-spatial processing is a student’s ability to perceive, analyze, and understand visual information and communicate with the teacher and peers.
English Language Proficiency for English (as a 2nd language) Learners

Students use their emerging, developing, or expanding 2nd language (English) to make meaning and engage with mathematics understandings. The student observations presented here will concentrate on students at the emerging, developing, and expanding levels of English language proficiency (ELP) as measured by the ACCESS for ELLs (CPL levels 2, 3, and 4) which describe most EL students. Recently arrived ELs at ELP 1 comprise <10% of the EL student population, and those who have achieved ELP 5 have exited the EL status and ESOL program services. It is expected that teacher’s will identify their student’s level of English proficiency to select the appropriate teacher actions.

<table>
<thead>
<tr>
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<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
<tbody>
<tr>
<td>Observations reveal emerging or developing comprehension or interpretation of mathematical explanations in English text and data with limited connections between prior mathematical learning and new mathematical concepts. (Interpretive Language Skills: Listening, viewing, and reading)</td>
<td>The teacher will support comprehension by providing temporary scaffolds that include: • making language visible by modeling formal definitions of mathematical terms and phrases to describe concepts, processes, and purpose, • making explicit connections to prior learning and student background and to real life (familiar context, language, family, peers, interests) • offering multimodal representations of mathematical discourse and reasoning and providing multiple opportunities for structured peer interaction and review of concepts.</td>
<td>The student will make sense of mathematical concepts, reasoning, discourse, and data by creating visual representation using sketches of graphs, tables, real-life models, and recording data from video demonstrations. The student will make connections to prior learning and background and transfer to new real-life problem situations by engaging in peer interaction using primary language (L1) and gestures when necessary to recall and discuss the relation of the mathematical concepts to something relevant in their lives.</td>
<td>Language Demand in Mathematics Lessons (LDML) tool at <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a> Framework for Analyzing Word Problems: Guiding Questions, Language Demands, and Tasks for Teachers – <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a> Multiple Mathematical Lenses: Task, Learning, Teaching, and Power and Participation (MML tool) <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a> (Access code: ELL14118)</td>
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<tr>
<td>Observations reveal emerging or developing language (imprecise) to describe in English their observations, clarify thinking, explain computational strategies, justify solutions, and explain underlying relationships for selected approaches and reasoning. (Expressive)</td>
<td>The teacher will support communication by providing temporary scaffolds that include: • teacher modeling of observational and comparative language, by strategic (probing) questioning and references to student work, • use of multiple</td>
<td>The student will record mathematical reasoning by increasingly using the following language structures in English: • generalized, abstract or multi-meaning noun groups to provide precision to descriptions, • observational and comparative language</td>
<td>Language Demand in Mathematics Lessons (LDML) tool at <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a> Framework for Analyzing Word Problems: Guiding Questions, Language Demands, and Tasks for Teachers – <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a></td>
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### Language Skills:
**Speaking and writing skills**

**Note:** Depending on students’ level of English language proficiency, they may use 1st language (L1) or gestures to clarify tasks orally and 2nd language (L2) to read text and make meaning. Code-switching between 1st and 2nd language may occur in peer interactions.

<table>
<thead>
<tr>
<th>Representations (numerical tables, graphs, algebraic expressions or images from a graphing calculator and models) to support sense-making, and</th>
<th>To demonstrate understanding of concept, functions, underlying relationships, and models,</th>
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<tbody>
<tr>
<td>• multiple opportunities for engaging peer interactions (partner work) to negotiate meaning using observational and comparative language,</td>
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<tr>
<td>• word bank with terms used to provide a strong rationale, justify, clarify,</td>
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<tr>
<td>• wait time, as needed, to process mathematical thinking in primary or 1st language before speaking (articulating) or writing in English (2nd language).</td>
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<tr>
<td>• visual data displays (drawings, software, demonstrations, tables, charts, etc.) to clarify approaches and solutions,</td>
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<td>• connectors to link sentences, to establish causality, and to clarify,</td>
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<tr>
<td>• reference devices to create cohesion,</td>
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<tr>
<td>• imperative verbs to establish a process or approach, and</td>
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<tr>
<td>• conditional conjunctions to propose future options.</td>
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<tr>
<th>Mathematical Routines</th>
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<tr>
<td>Multiple Mathematical Lenses: Task, Learning, Teaching, and Power and Participation (MML tool) <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a></td>
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### Observations reveal expanding comprehension and interpretation of mathematical explanations in English, such as problem-solving steps or evaluating a pattern or 

<table>
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<th>The teacher will support by providing temporary scaffolds that include:</th>
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<tr>
<td>• making language visible through repeated teacher modeling and think-</td>
</tr>
<tr>
<td>• language demand in mathematics lessons (LDML) tool at <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a></td>
</tr>
<tr>
<td>The student will make sense of word problems in English through using manipulatives, and creating visual representation, data charts, data graphs, and responding to listening</td>
</tr>
<tr>
<td>Framework for Analyzing Word Problems: Guiding Questions, Language</td>
</tr>
<tr>
<td>Interpretive Language Skills: listening, viewing, and reading</td>
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</tr>
<tr>
<td>Observations reveal expanding language to describe mathematical explanations in English. (Expressive Language Skills: Speaking and writing skills)</td>
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</tbody>
</table>
Additional Resources

Establish mathematics goals to focus learning.
- **5 Teaching Strategies to build math confidence**
  This website discusses 5 growth mindset strategies for students who lack math confidence.
- **10 Common Challenges Article**
  This article describes various ways teachers can make decisions based on their instructional goals.
- **Resources to Increase Students Interest**
  This is a website with different resources to increase student interest.

Implement tasks that promote reasoning and problem solving.
- **GSE Effective Instructional Practices Guide**
  This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices.
- **9 Strategies to Improving Problem-Solving**
  This website gives great insight to different techniques for effective questioning in the classroom.

Use and connect mathematical representations.
- **Visual Representations in Videos**
  A list of videos to provide teachers with intervention or remedial strategies of visual representations to support abstract learning.

Facilitate meaningful mathematical discourse.
- **10 Strategies to Enhance Students' Memory**
  While this website focuses primarily on literacy, the strategies involved can easily translate to teaching mathematics.
- **15 strategies for managing attention**
  This is a free website to provide strategies for enhancing attention or managing attention problems.
- **Academic Vocabulary Strategies**
  This website provides academic vocabulary strategies and resources to assist students with understanding mathematical language.
- **Multi-Sensory Instruction for Mathematics**
  10 ways to utilize multi-sensory instruction in the mathematics classroom.

Pose purposeful questions.
- **GSE Effective Instructional Practices Guide**
  This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices.
• Is Your Answer Reasonable?
  This lesson gives the teacher suggestions for guiding students in determining reasonableness.

Build procedural fluency from conceptual understanding.
• 8 Ways to Support Flexibility
  This is a free website to provide teachers with different math approaches to solving problems.

• Multi-Sensory Instruction for Mathematics
  10 ways to utilize multi-sensory instruction in the mathematics classroom.

Support efforts of learning in mathematics.
• Why are there so many Words in Math? Planning for Content-Area Math Instruction
  This article includes a method for incorporating Vocabulary into lessons.

• Three Reads Using a Problem Stem
  Provides definition of Three Reads, background information, and sample Three Reads lessons

• 9 Strategies to Improving Problem-Solving
  This website gives great insight to different techniques for effective questioning in the classroom.

Elicit and use evidence of student thinking.
• Formative Strategies Resources for Face to Face
  This is a free website to provide strategies to monitor students’ learning progress in a brick-and-mortar classroom.

• Formative Strategies Resources for Online Teaching
  This is a free website to provide strategies to monitor students’ learning progress in a remote or hybrid classroom.

• How to integrate effective feedback during instruction
  This website provides teachers with strategies to assist students with developing fluency by implementing effective feedback for students.

• How to implement aggressive monitoring in the classroom
  This website provides teachers with developing a strategy to monitor new learning as well as a method to elicit and provide feedback on calculations.
References

http://www2.edc.org/accessmath/resources/strategiesToolkit.pdf


Celedón-Pattichis, S., & Ramírez, N.G. (2013) Advancing mathematics education for ELs – Beyond good teaching (NCTM)


