Big Idea/ Topic

- Demonstrate knowledge of the definition of the derivative and the graphical interpretation.

Standard(s) Alignment

**MC.D.1** Students will demonstrate an understanding of the definition of the derivative of a function at a point, and the notion of differentiability.

a. Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.
b. Demonstrate an understanding of the interpretation of the derivative as instantaneous rate of change.
c. Use derivatives to solve a variety of problems coming from physics, chemistry, economics, etc. that involve the rate of change of a function.

Diagnostic Assessment

In the graph, the functions:

- \( f(x) = 2x \)
- \( g(x) = x^2 \)
- \( h(x) = 2^x \)

are shown. Look at all the graphs at \( x = 5 \).

Which graph is increasing at the fastest rate? Which graph is decreasing at the fastest rate?

Rank them from slowest to fastest.
Function at \( x = 4 \) | Rank Rate of Change (Slowest = 1 to Fastest = 3)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
</tr>
<tr>
<td>( g(x) )</td>
<td></td>
</tr>
<tr>
<td>( h(x) )</td>
<td></td>
</tr>
</tbody>
</table>

a) Explain why you ordered the functions as you did.

b) What is the rate of change for \( f(x) \) at \( x = 4 \)?

c) Why is it harder to evaluate the rate of change for \( g(x) \) and \( h(x) \) at \( x = 4 \)?

Instructional Design

Desmos Activity: **Instantaneous Rate of Change**

Engage

Jon is an avid biker. He leaves his house and travels south for some time and then turns around. He then heads north (past his house) until he gets to the store. The function:

\[
f(x) = 2x^2 - x \quad \text{from } [0,8]
\]

models how many kilometers Jon is **north** of his house after \( x \) minutes.

- \( x \) = number of minutes since Jon left his house
- \( f(x) \) = number of kilometers **north** of his house

We want to investigate how fast Jon is biking 4 minutes after he leaves his house.

Take a guess!

Teacher Moves:

If students have trouble formulating a hypothesis, probe them with questions about which direction Jon is running in. Is \( f(x) \), the number of km north Jon is from his home increasing or decreasing? This will help give students the idea of a positive slope or negative slope.
- **Synchronous:** Complete during a classroom discussion while pausing the activity to highlight student responses.
- **Asynchronous:** Introduce the problem to students in a virtual platform; this can be done via e-document or video. Allow students to share responses within the Desmos platform and provide feedback via the teacher dashboard. Additionally, students could use an audio/video to share. Provide feedback to individual student responses and highlight multiple strategies used by students.
- **Unplugged/Offline:** Provide the opening image for students to engage in the task. Have students share ideas through email/text/phone. Provide feedback to students and share other students’ ideas before engaging in the remaining sections.

**Explore**

Notice the graph of \( f(x) = .2x^2 - x \)

A secant line (a line that intersects the function at two points) is shown. The x-coordinate of the blue point is 4. The x-coordinate of the black point is 6.

What do you think would happen to the line shown if the black point were to move closer and closer to the blue point?

**Teacher Moves:** This could be a good time to reference back the definition of a secant and tangent line. Students are probably familiar with these terms with circles. A tangent line just touches a curve at a point. A secant line intersects two points on a curve.
Teacher Moves: Remind students that the function allows us to calculate the y-coordinates for the two points. From the two points, students should remember how to calculate the slope of a line. A useful mantra that students remember is “rise over run”.

Teacher Moves: Don’t rush this part of the activity. This is a key connection students need to realize to fully understand the definition of a derivative. Let students explore this part of the Desmos calculator. If students can realize how average rate of change (in this example: average velocity) approaches the instantaneous rate of change (in this example: instantaneous velocity) as alpha gets small, they will have a much deeper understanding of the definition of the derivative.
Teacher Moves: This is the reveal! Discuss each step of the difference quotient to make sure students understand the computational steps. Students should be able to relate the answer to their guesses (from the Desmos slider component) and envision a tangent line at the given point. This is the graphical representation of the definition of a derivative, which is so important for students to understand.

- **Synchronous**: Complete Desmos activity during synchronous learning, either face-to-face, virtual, or blended.
- **Asynchronous**: Give students time to complete the screens and provide feedback. Ensure that enough time is provided for students to participate and respond to your feedback and edit responses as needed.
- **Unplugged/ Offline**: Provide paper/electronic versions of the graphs and equations of screens 1-9. Allow students time to complete the work and submit through email/text or other means. Provide feedback and share with other students and provide access to other students’ thinking. Then, possibly assign the problem on screen 12 to check understanding (formative assessment).
Apply

A ball is thrown upwards from a roof top, 80m above the ground. It will reach a maximum vertical height and then fall back to the ground. The height of the ball from the ground is modeled by:

\[ f(x) = -16x^2 + 64x + 80 \]

where \( x \) = time in seconds after the ball was thrown
and \( f(x) \) = the height of the ball in meters

a) Determine the average rate of change (velocity) for the ball on the interval \([0,2]\).
b) Determine the instantaneous rate of change (velocity) for the ball at 1 second.

Teacher Moves: Relate this concept to a concept most students already understand. For example, if we make a 100-mile trip in 2 hours, our average velocity is 50 mph. However, that doesn’t mean that 30 minutes in the trip we were going 50 mph. This is the difference between average rate of change and instantaneous rate of change.

- **Synchronous** Complete Desmos activity during synchronous learning, either face-to-face, virtual, or blended.
- **Asynchronous** Using the teacher dashboard, unrestrict screens 4 through 9. Give students time to complete the screens and provide feedback. Ensure that enough time is provided for students to participate and respond to your feedback and edit responses as needed.
- **Unplugged/ Offline** Provide students with access to graph paper and allow students to engage in the questions presented on screens 4 through 9. Ask students to complete the questions and have them submit responses via email/text/phone. Provide feedback, share these responses with other students, and share other students’ responses with them.

Reflect

1) What are the key similarities and differences between finding the rate of change (slope) or a line vs. the rate of change of a nonlinear curve?

2) In what real life situations might we be concerned with the instantaneous rate of change for a function?

3) Students will create a Frayer Model graphic organizer to describe the **instantaneous rate of change**: What it IS, What it IS NOT, Examples, and NON-Examples.
**Synchronous: Think-pair-share.** First, students work independently to think about the questions posed and complete the Frayer Model. Next, students pair up and share their answers with each other (find disagreements for talking points). Finally, students engage in a large group discussion to discuss their answers. If you’re working synchronously online, you might explore your ability to have breakout rooms to allow students to work in groups.

**Asynchronous: Virtual Think-Pair-Share.** First, students work independently to answer the questions and attempt to complete the Frayer Model. If you’re able to group your students, you might consider having them work together to complete the Frayer Model.

**Unplugged/ Offline:** Provide students the first two questions and a blank template of a Frayer Model and instruct them to complete it with information about instantaneous rate of change.

<table>
<thead>
<tr>
<th>Evidence of Student Success</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formative Assessment Questions:</strong></td>
</tr>
<tr>
<td>- Can you find the average rate of change for a function on a given interval?</td>
</tr>
<tr>
<td>- Can you estimate the instantaneous rate of change for a function at a given x-value on the function by using values very close to the x-value?</td>
</tr>
<tr>
<td>- Can you think of how we could use limits to evaluate the instantaneous rate of change for a function at a given x-value?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Learning Supports</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Establish mathematics goals to focus learning.</strong></td>
</tr>
<tr>
<td>- Make instructions and expectations clear for the activities.</td>
</tr>
<tr>
<td>- Make explicit connections between current and prior lessons or units.</td>
</tr>
<tr>
<td><strong>Facilitate meaningful mathematical discourse.</strong></td>
</tr>
<tr>
<td>- Explicitly model and teach good “discussion board” etiquette.</td>
</tr>
<tr>
<td><strong>Pose purposeful questions.</strong></td>
</tr>
<tr>
<td>- Predetermine when you will call on the student or use the pause feature within the activities.</td>
</tr>
<tr>
<td>- Break class into small discussion groups to work collaboratively and then have groups report back to the whole group.</td>
</tr>
<tr>
<td><strong>Support productive struggle in learning mathematics.</strong></td>
</tr>
<tr>
<td>- Offer outlines and other scaffolding tools and share tips that might help students learn.</td>
</tr>
<tr>
<td>- Provide feedback using the feedback feature within activities and offer corrective opportunities.</td>
</tr>
<tr>
<td>- Consider the pacing of the lesson.</td>
</tr>
<tr>
<td>- Allow ample time for students to view screen 9. Students are often required to use the formal definition of a derivative and this screen should be a lasting visualization.</td>
</tr>
</tbody>
</table>
Elicit and use evidence of student thinking.

- Anticipate any misconceptions or questions students might have about the task, materials or technology. Proactively address them with readily available and accessible resources.
- Relate average and instantaneous rates of change to speeds in vehicles (students already have an understanding of the speed in which they are driving/riding in a vehicle).

<table>
<thead>
<tr>
<th>Engaging Families</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students can deepen their understanding of derivatives by completing the following activities asynchronously.</td>
</tr>
</tbody>
</table>
| □ **The Definition of the Derivative – GeoGebra**  
  This dynamic applet shows the transition of a secant line between two points to a tangent line at one point (limit as the distance between the two points gets very small). |
| □ **Derivatives Applet (integral-domain.org)**  
  This applet helps students see the relationship between the tangent line at a point, the first derivative, and the second derivative. |
| □ **Applet: Ordinary derivative by limit definition - Math Insight**  
  This applet helps students see the relationship between the secant line and the tangent line (limit as the distance between the two gets really small). |
| □ **Formal definition of the derivative as a limit (video)**  
  This video explains the definition of the derivative (with several examples). |
| □ **Derivative – Wolfram Alpha**  
  This is somewhat of a super calculator for derivatives. Provide a function and the calculator will display the derivative with graphical and written explanations. |