Big Idea(s)/ Topic(s)

- Develop an understanding of function and use functions to describe quantitative relationships.

Standard(s) Alignment

- **MGSE8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **MGSE8.F.3** Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- **MGSE8.F.2**. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

Diagnostic Assessment

1. If \( m = 12.5\% \), circle all the statements below that are true for the expressions \( 2m \).
   
   A. \( 2m = 17.5\% \)
   B. \( 2m = 12.52\% \)
   C. \( \frac{25}{100} \)
   D. \( 2m = 2 \) meters
   E. \( 2m = 25\% \)
   F. After investing $125, Louisa lost $31.25 in a day.

   Explain your reasoning for each circled statement.

2. Below are three equations and three graphs, representing a total of six different functions. Put them in ascending order (from least to greatest) according to the rate of change.

   A. \( y = 3x \)
   B. \( y = -2x + 5 \)
   C. \( y = x + 5 \)
   D. \( y = \frac{25}{100} \)
   E. \( \frac{25}{100} \)
   F. After investing $125, Louisa lost $31.25 in a day.
Instructional Design

Desmos Activity: Grade 8: Functions

Overview: In this task, students will determine whether a function is increasing or decreasing based on whether its rate of change is positive or negative. Students will explain in their own words how the graph of a linear function relates to its rate of change and initial value.

Materials:
- Access to the diagnostic assessment
- Access to the Desmos activity (or a printout)
- Graph paper
- Sticky notes
- Create a discussion board for this learning activity using platforms such as Kialo Edu or Yo Teach for asynchronous learners.

Engage

Teacher Moves

Present the screen an inform students that all three graphs have the same scale. Ask students "how are the graphs different?" Engage in a conversation about the variation before defending Diego or Mai.
Give students 1–2 minutes of quiet work time, followed by a rich whole-class discussion.

Sample Responses

Possible Responses
"In the first graph, the line decreases when we read left to right. That means it has a negative slope. The second line stays horizontal the entire time, so it must have a slope of zero. The third graph increases as we read left to right, so it has a positive slope. That means the slopes are ordered from least to greatest.
On the other hand, the y-intercept of the graph on the left is positive and higher than the second graph. The y-intercept of the last graph is negative, so the y-intercepts are ordered from greatest to least.
Alternatively, Mai may just be looking at the left side of the graphs where they "start" while Diego is looking at the right side of the graph where they "end up."

**Teacher Moves**

Present the screen and remind students. The time that Jada spent mowing lawns, is in a functional relationship with the amount of money, that Jada has earned.

**Sample Responses**

We can choose to think of time as a function of money, or vice versa.

\[
m = 7t \quad \text{if } t \text{ is independent}
\]

\[
t = \frac{m}{7} \quad \text{if } m \text{ is independent}
\]
**Synchronous:** After assigning the Desmos, click the pacing icon in the teacher dashboard and select screen 1. Next, click the orange option to restrict to screen one. (Please note that the pages that students will be able to freely access will be highlighted in orange.). Follow Teacher Moves and Sample Responses and use Desmos Snapshots to highlight student work. Restrict to screens two and three and follow the guidance in Teacher Moves. It is during these screens you also want to solidify independent, dependent variables and proportional reasoning concepts.

**Asynchronous:** Group students according to their diagnostic assessment. Use the Thumbs Mode and Overlay Mode in Desmos to determine students' knowledge of proportional relationships as it relates to functions, independent and dependent variables. Use Snapshots to create a discussion.

**Teacher Moves**

Present the screen and remind students. If students get stuck consider asking, “What could we do to the scale on the axes to see the constant rate of change on each graph and accurately compare them?” (Use the same scale on each axis, or graph both axes using the same length to represent 1 unit.)

**Activity Synthesis Screen 2 & 3**

The purpose of this discussion is for students to understand that proportional relationships are functions and to connect the parts of functions to what they know about proportional relationships. Select previously identified students to share their equations and graphs. Sequence student work so that at least one example of each representation is shown for each of the problems, starting with the most common representation. Display their responses for all to see. Consider asking some of the following questions to help students make connections between the different representations:

- “For the first problem, if we wanted to know how many hours Jada needs to work to make a certain amount of money, which equation would make more sense to use? Why?” (t = \( \frac{1}{i} \) M, because in that equation, time worked, t, is expressed as a function of money earned, M.)
- “For the second problem, when would we want to use the equation \( f = 3y? \)” (When we know the number of yards and need to calculate the number of feet.)
- “How do we know that each of these situations are represented by functions?” (For each valid input, there is only one output. For example, no matter which equation I use for the relationship between feet and yards, a specific number of feet will always equal the same number of yards.)

**Sample Responses**

1. The value of a measurement in yards, \( y \), is in a functional relationship with the value \( f \) of that same measurement in feet. We can choose to think of \( y \) as a function of \( f \), or vice versa.
2. We write \( f = 3y \) if we choose \( f \) as the dependent variable and \( y \) as the independent variable, or \( y = \frac{f}{3} \) if we choose \( y \) as the dependent variable and \( f \) as the independent variable.
thread if many students have inappropriate answers. It may be best to provide a prerecorded video explaining screens 1 through 3 after the student completes them.

- **Unplugged/ Offline**: Provide the image and question from screen 1 and allow students time to answer the questions. Address any misconceptions, follow Teacher Moves, and allow students time to address your feedback. Don’t forget to “cross share” student work (online and offline). Next provide images and questions from screen two and three. Remind students that graphs and equations can be used to better understand what’s happening in a function.

**Explore**

**4 Four Tanks of Water**

**Teacher Moves**

Use the teacher dashboard to identify students who may need additional support before completing the task.

Consider customizing Desmos to have one question per screen.

**Activity Synthesis**

Consider asking some of the following questions to begin the discussion:

- “For the second problem, what in the equation tells you that the slope is decreasing? Increasing?” (Decreasing: the slope in the equation is negative; Increasing: the slope in the equation is positive.)
- “For the third problem, what is similar between the equation in this problem and the decreasing equation in the previous problem?”

(Both slopes are negative.)
- “For the last problem, what in the graph tells you that Tank D is draining out? What would a graph that has a tank filling up look like? What would be different?” (I know it is draining out because the graph is going down from left to right. If the tank were filling up, the graph would be going up from left to right.)

Tell students that a linear function can always be represented with an equation of the form \( y = mx + b \). The slope of the line, \( m \), is the rate of the change of the function and the initial value of the function is \( b \).

**Sample Responses**

1. Tank B started out with more water. Tank A started out with 200 gallons.
2. \( B = 400 + -5t \) or \( B = 400 - 5t \)
3. Tank A is filling up, and Tank B is draining out. As time goes on, corresponding to larger values of \( t \), the value of \( A \) gets bigger, but the value of \( B \) gets smaller.
4. Draining out. As \( t \) increases, the value of \( C \) decreases, since we are subtracting larger values from 800. In short, it is because we are subtracting multiples of \( t \) instead of adding them that we can quickly see that Tank C is draining.
5. Draining out. As time increases, the value of \( D \) goes down.

The purpose of this activity is to put features of an equation representing a function into context.
Students will start with two functions: one that represents a tank being filled up and another that represents a tank being drained out. Students will be asked to determine which equation represents each situation. This gives students the opportunity to connect initial value and slope to the general form of the linear equation and to the fact that linear relationships are functions.

- **Synchronous**: Restrict to screen 4, consider printing out the pre-filled graphic organizer on the next screen for students with accommodations. Follow Teacher Moves and scaffold concepts as necessary.
- **Asynchronous**: Create a collaborative document and house a blank template with tanks labeled in each quadrant. Have students work together to state noticing, givens and wondering under each. Ensure that enough time is provided for students to participate and respond to your feedback and edit responses as needed.
- **Unplugged/ Offline**: Provide paper/electronic versions of screens four and a pre-filled graphic organizer from screen 5. For face-to-face students, arrange students in groups of 2 to 4. Give students 3–5 minutes of quiet work time and then time to share their responses with their partner and reach an agreement on their answers. Encourage partners to talk about specific parts of the graph and equation that indicate whether the tank is filling up or draining out. Follow with a whole-class discussion.

**Apply**

**Teacher Moves**

Present the screen to students.

**Sample Responses**

Answers vary.

- Sample response: It takes 80 minutes for Tank B to drain, because \( \frac{400}{5} = 80 \). The tank is 70% full after 30% of that time has elapsed. The tank is 30% full after 70% of that time has elapsed.
- The point (1.5, 10.5) is 30% of the way from (0,15) to (5,0), because \( 0.3 \times 5 = 1.5 \) & \( 0.7 \times 15 = 10.5 \). The point (3.5, 4.5) is 70% of the way from (0,15) to (5,0), because \( 0.7 \times 5 = 3.5 \) & \( 0.3 \times 15 = 4.5 \).
- The point (4.5, 5.3) is 30% of the way from (3,5) to (8,6). The coordinates are 5 units apart, because 8 - 3 = 5. 30% of 5 is 1.5 and 3 + 1.5 = 4.5. The y -coordinates are 1 unit apart, because 6 - 5 = 1. 30% of 1 is 0.3 and 5 + 0.3 = 5.3.
  The point (6.5, 5.7) is 70% of the way from (3,5) to (8,6). 70% of 5 is 3.5 and 3 + 3.5 = 6.5. 70% of 1 is 0.7 and 5 + 0.7 = 5.7.
Noah is depositing money in his account every week to save money. The graph shows the amount he has saved as a function of time since he opened his account.

Elena opened an account the same day as Noah. The amount of money $E$ in her account is given by the function $E = 8w + 60$, where $w$ is the number of weeks since the account was opened.

**Teacher Moves**

Display the screen. Highlight student responses.

**Activity Synthesis**

Display the graph of Noah’s savings over time and the equation for the amount of money in Elena’s account for all to see. Select students previously identified to share their responses.

Consider asking the following questions to help the student make connections between the different representations:

- “How did you determine the amount Noah saved in a year?” (I used the graph to figure out that Noah saves $5 each week and multiplied that by 52 weeks.)
- “What equations could you use to solve the last question?” (I could use the equation $N = 60 + 5w$ for the amount of money in Noah’s account after $w$ weeks. When $w = 52$, Noah has $320. If I solve the equation $320 = 8w + 60$ for $w$, I would know how many weeks it would take Elena to have $320 in her account.)
- “How could you solve the last question without using an equation?” (I could extend the graph out to 52 weeks and plot the value of each account over the year.)

**Sample Responses**

1. They are the same. At the left edge of the graph, representing the time when they opened the accounts, Noah had $60. When $t$ is 0, the money in Elena's account when it was opened is found by $E = 8(0) + 60$, so she also had $60.

2. Elena is saving money at a faster rate. Every 2 weeks, Noah's account increases by $10 while Elena's account goes up by $8 each week, so she makes $16 in two weeks.

3. Noah will save $260 over a year in addition to the $60 he opened the account with, since he saves $10 every 2 weeks and there are 52 weeks in the year. It will take Elena just 33 weeks to save the same amount since she also started with $60 ($\frac{260}{8} = 32.5$, so rounding up, it will take 33 weeks).
How can we tell if a linear function is increasing from an equation? From a graph?

**Sample Responses**

(In a linear equation \( y = mx + b \), if \( m \) is positive, the linear function is increasing. In a graph, if the line is going up from left to right, then the function is increasing.)

(If the graph of the function crosses the vertical axis above 0, then the initial value is positive. In the equation, \( y = mx + b \) of a linear function, the \( b \) is positive when the initial value is positive.)

**Teacher Moves**

Explain that an equinox is the day when there is an approximately equal amount of daylight and darkness. In a certain city in France, they gain 2 minutes of daylight each day after the spring equinox (usually in March), but after the autumnal equinox (usually in September) they lose 2 minutes of daylight each day.

**Sample Responses**

d

**Sample Responses**

b

**Sample Responses**

Graph A does not make sense because there is a constant amount of daylight. Graph C does not make sense because it goes through the origin, meaning it started with 0 minutes of daylight.

Is the math sentence below always, sometimes, or never true?

The rate of change will always be multiplied by the independent variable.

**Teacher Moves**

Have students share their thinking aloud.

**Sample Responses**

Always True
Remember to explain to students that an equinox is the day when there is an approximately equal amount of daylight and darkness (screen 8).

- **Synchronous**: Present and allow students to connect their work with linear equations to functions. Pace students as needed. Identify students who use different methods to answer the questions. Highlight and engage in rich discussion following guidance above.
- **Asynchronous**: Allow students to answer screen 5 in a Flipgrid. Add the “Synthesis” screen as a poll or on the collaborative document created earlier. Ensure that enough time is provided for students to participate and respond to your feedback and edit responses as needed.
- **Unplugged/ Offline**: Provide students with access to graph paper and allow students to engage in the questions presented on screens 5 through 11. Ask students to complete the questions and have them submit responses via email/text/phone. Provide feedback, share responses with other students, and share other students’ responses with them.

### Reflect

<table>
<thead>
<tr>
<th>3-2-1:</th>
<th>List three things you learned in this lesson, two things you want to know more about, and one thing you are confused about.</th>
</tr>
</thead>
</table>

**Teacher Moves**

- Consider editing the graph and change either of the questions to your learning targets.
- Have students share out their 3-2-1 summarizer or customize and allow students to see each other's responses.

### Ticket out the Door: 3-2-1 (on Desmos screen)

- **Synchronous**: Make sure pacing has stopped and allow students time to type and read other students’ responses.
- **Asynchronous**: If possible, make sure pacing has stopped. Students will type and read other students' responses on screen 12.
- **Unplugged/ Offline**: Students can write their answers on a sticky note or in their math journals.

### Evidence of Student Success

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. Evidence of fluency can be assessed throughout the lesson. Have students answer the posed question below independently.

1. Two car services offer to pick you up and take you to your destination. Service A charges 40 cents to pick you up and 30 cents for each mile of your trip. Service B charges $1.10 to pick you up and charges cents for each mile of your trip.
a. Match the services to the Lines l and m.

b. For Service B, is the additional charge per mile greater or less than 30 cents per mile of the trip? Explain your reasoning.

- Can students determine whether a function is increasing or decreasing based on whether the rate of change is positive or negative?

- Can students explain in my own words how the graph of a linear function relates to the rate of change and initial value?

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**Student Learning Supports**

**Establish mathematics goals to focus learning.**
- Make instructions and expectations clear for the activities.
- Make explicit connections between current and prior lessons or units.

**Facilitate meaningful mathematical discourse.**
- Explicitly model and teach good “discussion board” etiquette.

**Pose purposeful questions.**
- Predetermine when you will call on the student or use the pause feature within the activities.
- Break class into small discussion groups to work collaboratively and then have groups report back to the whole group.

**Support productive struggle in learning mathematics.**
- Offer outlines and other scaffolding tools and share tips that might help students learn.
- Provide feedback using the feedback feature within activities and offer corrective opportunities.
- Consider the pacing of the lesson.
Possible Remediation or Extension Modules:

- What is a function:
  https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:functions/x2f8bb11595b61c86:evaluating-functions/v/what-is-a-function?modal=1
- Equation of a line Manipulation:
  https://gpb.pbslearningmedia.org/resource/mgbh.math.ns.mangraph/manipulating-graphs/
- Equation vs. Function:
- Obtaining a Function from an Equation:
  https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:functions/x2f8bb11595b61c86:functions-and-equations/v/create-function-from-equation?modal=1
- Turtle Crossing:
  https://teacher.desmos.com/activitybuilder/custom/5ddbf9ae009cd90bcdeadd7?collections=5da6476150c0c36a0caf8fb
- Put It All Together:

Engaging Families

Students can deepen their understanding of functions by completing the following virtual class here or try out this card sort. Students may find it helpful to review the concepts alongside their parents, siblings, or friends at home.

Students may also find it helpful for all household members to watch this quick video and determine what is most important. The subsequent video was found on Georgia Home Classroom site, many more resources are located on this site. Feel free to peruse.
Diagnostic Assessment
Grade 7 – Functions

1. If \( m = 12.5\% \), circle all the statements below that are true for the expressions \( 2m \).

   A. \( 2m = 17.5\% \)
   B. \( 2m = 12.52\% \)
   C. \( \frac{25}{100} \)
   D. \( 2m = 2 \) meters
   E. \( 2m = 25\% \)
   F. After investing $125, Louisa lost $31.25 in a day.

Explain your reasoning for each circled statement.

2. Below are three equations and three graphs, representing a total of six different functions. Put them in ascending order (from least to greatest) according to the rate of change.

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   B. \( y = -2x + 5 \)
   C. \( y = x + 5 \)
   D. \( y = 2x \)
   E. \( y = -x + 5 \)
   F. \( y = x + 5 \)