### Big Idea/ Topic

- Use properties of rational and irrational numbers to rewrite expressions involving square roots to solve problems.

### Standard(s) Alignment

Use properties of rational and irrational numbers.

**MGSE9-12.N.RN.3** Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.

### Diagnostic Assessment

**Rational vs. Irrational Numbers Justified True/False Statements**

Students will answer diagnostic questions about rational and irrational numbers, along with perfect and imperfect squares.
Instructional Design

Desmos Activity: Square Dance !!!

Engage

1 Meet Blue. He's a pe...

Teacher Moves

Consider letting students work at their own pace on screens 1-4. Consider having a classroom discussion on Screen 5.

2

Teacher Moves

Students will be invited to share what they notice and wonder.

Questions for thought:
Beyond this interactive, is it possible to create a square with side lengths that are not whole numbers?
Given only the area, how could one deduce the side length of a square? What is the exact length of each square's diagonal?

- **Synchronous**: Complete during a classroom discussion while pausing the activity to highlight student responses.
- **Asynchronous**: Introduce the problem to students in a virtual platform; this can be done via e-document or video. Allow students to share responses within the Desmos platform and provide feedback via the teacher dashboard. Additionally, students could use an audio/video to share. Provide feedback to individual student responses and highlight multiple strategies used by students.
- **Unplugged/ Offline**: Provide the image on slide two for students to engage in the task. Have students share ideas through email/text/phone. Provide feedback to students and share other students’ ideas before engaging in the remaining sections.

Explore

What did you notice about Blue’s side length and his total area?

Teacher Moves

Students might make connections to:
- finding the area of quadrilaterals.
- squares having equal side lengths
- the area is the side length multiplied by itself
- other...

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Blue is a type of square that always has INTEGER side lengths.

Imagine Blue’s area is 100 square units (100 \( \text{u}^2 \)). How many units long is one of his sides?

**Sample Responses**

10

The side length of a square with area 100 is 10. We call 10 the SQUARE ROOT of 100.

Similarly, the square root of 25 is 5 because Blue’s side length is 5 when his area is 25.

What is the square root of 81?

**Teacher Moves**

Students might think that rational numbers are only whole numbers. Use this opportunity to explore perfect squares that have decimals and fractions. For example:

\[
\sqrt{0.49} = 0.7 \\
\sqrt{\frac{9}{25}} = \frac{3}{5}
\]

This is the square root symbol: \( \sqrt{\text{ }} \)

You may have noticed that the square root of Blue’s area is equal to his side length.

**Teacher Moves**

Students might think that rational numbers are only whole numbers. Use this opportunity to explore perfect squares that have decimals and fractions. For example:

\[
\sqrt{0.49} = 0.7 \\
\sqrt{\frac{9}{25}} = \frac{3}{5}
\]

**Teacher Moves**

A thumbs up picture will appear when all values are placed correctly. Each square root is a perfect square.
● **Synchronous**: Complete Desmos activity during synchronous learning, either face to face, virtual, or blended.

● **Asynchronous**: Using the teacher dashboard, unrestrict screens three through eight. Give students time to complete the screens and provide feedback. Ensure that enough time is provided for students to participate and respond to your feedback and edit responses.

● **Unplugged/Offline**: Provide printed materials for students to draw squares on a grid and to place radicals on a number line.

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**Teacher Moves**

Students can be prompted to think of .25 as $\frac{1}{4}$ and .16 as $\frac{16}{100}$ to see them as perfect squares in a different manner.

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**Apply**

9 Group all the equivalent...

**Teacher Moves**

Remind students that feedback is available on the next slide. Encourage students to amend their responses based on the feedback on the following slide.

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10

**Teacher Moves**

Students may ask about the square grids under the radical. This is to strengthen the geometric and algebraic association of square roots.
Imagine Purple's area is 40 square units.
What do you think her side length could be?

**Teacher Moves**

If students pick terminating decimals for side lengths have them square those side lengths to try for an area of 40.

Talk about why approximations like $6.3245^2$ still not square to equal 40. The conversation may deepen as students arrive at the conclusion that no rational, terminating decimal side length will square to equal 40.

**Sample Responses**

Accurate decimal approximations may include 6.32, 6.324, 6.3245 and 6.3246

Students struggling with the concept of area may respond with 10 or 20.

**Teacher Moves**

1) The square turns blue when its side lengths are whole numbers.
2) The graphing calculator limits the decimal notation to 11 places (6.32455632034),

However, all areas are rounded to 2 decimal places for this slide.
1) In the left column, enter a whole number side length that is less than $\sqrt{40}$.

2) In the middle column, enter a side length that is approximately $\sqrt{40}$.

3) In the right column, enter a whole number side length that is greater than $\sqrt{40}$.

**Teacher Moves**

Look for how students round their decimal answers and talk about it as a class.

Questions to consider:

1) Why did some students round to a whole number, one decimal place, two decimal places...?
2) Did students use 6 and 7 for their whole number sides lengths less than and greater than $\sqrt{40}$? Why or why not?
3) Why is $\sqrt{40}$ not a whole number?

**Sample Responses**

- side length with area less than $\sqrt{40}$ : 6
- side length with area about equal to $\sqrt{40}$ : 6.3
- side length with area greater than $\sqrt{40}$ : 7

Correct range for side length is between 6.25 and 6.45, inclusive.

**Student Supports**

The table cells use TEXT INPUT to compel students to use decimals to approximate a side length with an area of 40.

**14**

- $\sqrt{6} \approx 6$
- $\sqrt{40}$ is between 6 and 7.
- $\sqrt{49} = 7$

The $\sqrt{40}$ is between 6 and 7.

How can we know that?

**Teacher Moves**

Discuss:

1) 6.32455532034...
2) The properties of irrational numbers
For squares with areas of 5, 32, 70, and so on, we’re not able to find EXACT decimals for their side lengths.

This is because finding an exact decimal representation for \( \sqrt{5} \), \( \sqrt{32} \), and \( \sqrt{70} \) is impossible.

They are examples of IRRATIONAL NUMBERS.

1. Their decimals don’t repeat
2. Their decimals don’t terminate
3. They cannot be written as a ratio of two integers

The best we can do is find decimal approximations.

**Teacher Moves**

You may want to talk about imperfect squares as irrational numbers. The conversation can also extend into other types of irrational numbers, like \( \pi \) and \( e \).

**Teacher Moves**

Students will slide the slider to reveal imperfect squares within the desired ranges.

**Teacher Moves**

If students don’t see a correct message appear, encourage them to be as precise as possible when placing the points. \( \sqrt{70} \) is the most sensitive value. Encourage students to place that value last if they are having difficulty.

**Teacher Moves**

Students might need to double check the precise placement of \( \sqrt{225} \).

- **Synchronous**: Complete Desmos activity during synchronous learning, either face-to-face, virtual, or blended.
- **Asynchronous**: Using the teacher dashboard, unrestrict screens nine through 18. Give students time to complete the screens and provide feedback. Ensure that enough time is provided for students to participate and respond to your feedback and edit responses as needed.
- **Unplugged/Offline**: Provide students with access to graph paper and allow students to engage in the questions presented on screens nine-18. Ask students to complete the questions and have them submit responses via email/text/phone. Provide feedback, share these responses with other students, and share other students’ responses with them.
Reflect

19 Share your general strategy for placing the eight points.

Teacher Moves
Students might be able to come up with informal ways to describe rational numbers, irrational numbers, and square roots. Help them formalize their thinking.

20 Group all the equivalent... (Diagram)

Teacher Moves
Remind students that feedback is available on the next slide. Encourage students to amend their responses based on the feedback on the following slide.

21 (Diagram)

Teacher Moves
Students may ask about the square grids under the radical. This is to strengthen the geometric and algebraic association of square roots.

- **Synchronous: Teach another student.** Students will share their general strategy for estimating irrational numbers with another student. Students will also share their strategy for completing a card sort with irrational values.
- **Asynchronous: Virtual Think-Pair-Share.** Students work independently to estimate irrational numbers and to complete the irrational number card sort.
- **Unplugged/ Offline:** Provide students a printed matching activity based on the card sort. Encourage students to reflect on their strategies.

Evidence of Student Success

Formative Assessment Questions:
- What is a perfect and imperfect square?
- Can you estimate the value of an imperfect square?
- Can the product of irrational numbers yield a rational product?
- Give an example.
- How can rewriting perfect square decimals, like .36 as fractions help in rewriting the perfect square (as a decimal or fraction)?
- Which number set is larger: perfect or imperfect squares?

Student Learning Supports

Establish mathematics goals to focus learning.
- Make instructions and expectations clear for the activities.
- Make explicit connections between current and prior lessons or units.
Facilitate meaningful mathematical discourse.
- Explicitly model and teach good “discussion board” etiquette.

Pose purposeful questions.
- Predetermine when you will call on the student or use the pause feature within the activities.
- Break class into small discussion groups to work collaboratively and then have groups report back to the whole group.

Support productive struggle in learning mathematics.
- Offer outlines and other scaffolding tools and share tips that might help students learn.
- Provide feedback using the feedback feature within activities and offer corrective opportunities.
- Consider the pacing of the lesson.

Elicit and use evidence of student thinking.
- Anticipate any misconceptions or questions students might have about the task, materials or technology. Proactively address them with readily available and accessible resources like the real numbers Venn diagram below, or these notes from Math is Fun.

![Real numbers Venn diagram](image)

Engaging Families
- Families can explore perfect and imperfect squares up to 100 with this engaging, interactive activity: Color the Radicals Activity
- Here is a helpful blog post about that compliments this lesson with more teaching ideas.
- Clothesline cards can be used to manipulate and sequence radicals and whole numbers. Students can match equivalent values and estimate irrational imperfect squares when sequencing the cards. Here is a relevant set of clothesline cards: http://bit.ly/squaredancecards.