Big Idea(s)/Topic(s)

- Students will prove theorems about lines and angles.
- Students will prove theorems about triangles.
- Students will prove theorems of parallelogram.
- Students will use similarity and congruence to prove theorems.

Standard(s) Alignment

Georgia Standards for Excellence:

- **MGSE9-12.G.CO.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

- **MGSE9-12.G.CO.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

- **MGSE9-12.G.CO.11** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Standards for Mathematical Practice:

- Make sense of problems and persevere in solving them.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Look for and make use of structure.

Diagnostic Assessment

The first slide of the Desmos activity is a warm up (card sort) meant to serve as a diagnostic assessment.
## Instructional Design

### Directions for Content Developers

**Desmos Activity Link:** [Proofs and Puzzles](#)

**Big Idea:** Students will understand how proofs are used to create theorems and postulates.

**Materials Needed:**
2. Printed materials for offline students.

### Engage

#### Teacher Moves

**2 Engage: Proofs and ...**

Use this graphic organizer to introduce the different types of proofs.

**3 Engage: Example of ...**

Use this graphic organizer to introduce the different types of proofs.

**4 Engage: Example of ...**

Use this graphic organizer to introduce the different types of proofs.

- **Synchronous:** The teacher will use this graphic organizer to introduce the different types of proofs to the students. The teacher can print it off for students or let students copy down the notes/ organizer.

- **Asynchronous:** The teacher will provide the desmos.com student link to students so that they can access the lesson at home. Students will use this page as notes.

- **Unplugged/ Offline:** Provide students with printable version of this lesson. Have students share ideas through email, text, photos, and/ or scan document applications.

A **Printable Version of Lesson** can be located in [Appendix A](#).
5 Explore: Angle Puzzle...

Click an angle to reveal its measure.
Then determine the rest of the angles using as few reveals as you can.

Teacher Moves

Consider using pacing to restrict students to Screens 1–3.

Sample Responses

Angle measures (clockwise from the top):
138°, 42°, 138°, 42°

6 Explore: Angle Puzzle...

Click an angle to reveal its measure.
Then determine the rest of the angles using as few reveals as you can.

Teacher Moves

This is a great place to check student progress. Use the summary view in the dashboard to identify students who may need additional support.

Sample Responses

Angle measures (from top to bottom):
45°, 45°, 45°, 135°

7 Explore: Angle Puzzle...

Click an angle to reveal its measure.
Then determine the rest of the angles using as few reveals as you can.

Teacher Moves

This is a great place to check student progress. Use the summary view in the dashboard to identify students who may need additional support.

Sample Responses

Angle measures (from top to bottom going left to right):
38°, 82°, 60°, 38°, 60°
Click an angle to reveal its measure.

Then determine the rest of the angles using as few reveals as you can.

Note: The square symbol indicates a right angle (90°).

**Teacher Moves**

This is a great place to check student progress. Use the summary view in the dashboard to identify students who may need additional support. Consider skipping Screens 7–8 if you are running out of time.

Consider using pacing to restrict students to Screens 7–10.

**Sample Responses**

Angle measures (from top to bottom going left to right):

49°, 75°, 105°, 56°, 34°, 41°

**9 Explore: Example of Two-Column Proofs**

Explain how a Two-Column Proof is like a puzzle.

**Teacher Moves**

Have students read the proof and point out the statements and reasons and how they go together.

**Sample Responses**

Students’ responses will vary.

**Student Supports**

The teacher can list out the statements and reasons out as steps to help students see the structure of the proof.

- **Note:** The teacher will talk to students about how a two column proof is set up by using the provided image. The teacher will then ask students to compare a two column proof to a puzzle.
Apply:

10 Apply Perpendiculars...

**Step 1:** Mark the picture according to what is given in the problem. (Use the sketch tool to mark your picture).

**Step 2:** List out the given information.

**Teacher Moves**

The teacher can use this slide as guided practice. On Step 7, students are required to list out the correct theorem that goes along with the reason.

**Sample Responses**

- Side-Angle-Side Theorem

**Student Supports**

- Have students work in pairs to help struggling students with the problems.

**Note:** The teacher can use this slide like a guided practice problem. Students will read through the steps to help them understand how each step reveals another piece of the puzzle or proof.

11 Apply: Proof of the S...

**Write a two-column proof that proves the sum of the angles in a triangle is 180°.**

**Teacher Moves**

The reasons are formatted so that the students get the correct order. The program will notify them and the teacher if the order is correct or not. Have students work in pairs for the remainder of the lesson.

**Sample Responses**

The reasons are formatted so that the students get the correct order. The program will notify them and the teacher if the order is correct or not.

**Student Supports**

- Have students work in pairs to help struggling students with the problems.
Put the Reasons in the correct order.

Teacher Moves

The teacher will introduce the problem and ask students to put the reasons in the correct order. Make sure that students understand how to reorganize the different reasons. Have students work in pairs for the remainder of the lesson.

Sample Responses

The reasons are formatted so that the students get the correct order. The program will notify them and the teacher if the order is correct or not.

Student Supports

Have students work in pairs to help struggling students with the problems.

Note: The teacher should instruct students to reveal an angle and have students fill in the missing angle measurements or pieces. The teacher should then give students time to answer the puzzle and share out their answers with the class.
Put the Reasons in the correct order.

Teacher Moves

The reasons are formatted so that the students get the correct order. The program will notify them and the teacher if the order is correct or not. Have students work in pairs for the remainder of the lesson.

Sample Responses

The reasons are formatted so that the students get the correct order. The program will notify them and the teacher if the order is correct or not.

Student Supports

Have students work in pairs to help struggling students with the problems.

Put the Reasons in the correct order.

Teacher Moves

The reasons are formatted so that the students get the correct order. The program will notify them and the teacher if the order is correct or not. Have students work in pairs for the remainder of the lesson.

Sample Responses

The reasons are formatted so that the students get the correct order. The program will notify them and the teacher if the order is correct or not.

Student Supports

Have students work in pairs to help struggling students with the problems.

16 Use this previous pr…

Teacher Moves

The teacher will ask students to design a flowchart by using the embedded link.

Sample Responses

Students' responses will vary.

Student Supports

Have students work in pairs.

● Note: The teacher should instruct students to create a flowchart proof using the handout in Appendix B.
Teacher Moves

The teacher will ask students to design a flowchart by using the embedded link.

Sample Responses

Students’ responses will vary.

Student Supports

Have students work in pairs.

Reflect

Teacher Moves

The teacher can use the Reflection Activity as a Ticket out the Door.

Sample Responses

Students’ responses will vary.

● **Synchronous:** Teacher will instruct students to fill out the Reflection Activity as a Ticket Out the Door.

● **Asynchronous:** The teacher will provide the desmos.com student link to students so that they can access the lesson at home. Students will complete this slide on their own devices. The teacher can use the dashboard to monitor student’s progress and provide feedback to the students.

● **Unplugged/ Offline:** Provide students with a printable version of this lesson. Have students share ideas through email, text, photos, and/ or scan document applications.

Evidence of Student Success

Formative Assessment Questions:

- How do I prove geometric theorems involving lines, angles, triangles, and parallelograms?
- How do I know which method to use to prove two triangles congruent?
- How do I know which method to use to prove two triangles similar?
### Student Learning Supports

**Establish mathematics goals to focus learning.**
- Make instructions and expectations clear for the activities.
- Make explicit connections between current and prior lessons or units.

**Facilitate meaningful mathematical discourse.**
- Explicitly model and teach good “discussion board” etiquette.

**Pose purposeful questions.**
- Predetermine when you will call on the student or use the pause feature within the activities.
- Break class into small discussion groups to work collaboratively and then have groups report back to the whole group.

**Support productive struggle in learning mathematics.**
- Offer outlines and other scaffolding tools and share tips that might help students learn.
- Provide feedback using the feedback feature within activities and offer corrective opportunities.
- Consider the pacing of the lesson.

**Elicit and use evidence of student thinking.**
- Anticipate any misconceptions or questions students might have about the task, materials or technology. Proactively address them with readily available and accessible resources.

### Additional Learning Support Strategies

- Allow students to work in pairs on this lesson.
- Provide students with the graphic organizers that are in the printable version of this lesson.

### Engaging Families

Students can deepen their understanding of Proofs by completing the following activities. Students may find it helpful to review the concepts alongside their parents, siblings, or friends at home.

- Math is Fun: Geometry Glossary- [https://mathsisfun.com/geometry/index.html](https://mathsisfun.com/geometry/index.html)
- Virtual Nerd: Congruence Proofs- [https://virtualnerd.com/texasteks/teksgeometry/6](https://virtualnerd.com/texasteks/teksgeometry/6)
Diagnostic: Warm Up Card Sort

*Cut cards out

- **Vertical Angles Theorem:** Vertical angles are congruent.
- **Exterior Angle Theorem:** An exterior angle of a triangle is equal to the sum of the two opposite interior angles.
- **Incenter Theorem:** When three interior angle bisectors of a triangle meet at a single point.
- **Corresponding Angles Postulate:** If two parallel lines are cut by a transversal, then corresponding angles formed by the transversal are congruent.
- **Isosceles Triangle Theorem:** If two sides of a triangle are congruent, then the angles opposite those sides are also congruent.
- **Altitudes Theorem:** When three altitudes intersect inside of a triangle and meet at a single point.

**Central Theorem:** The point where the three medians intersect.

**Triangle Midsegment Theorem:** If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and half its length.
Engage: Proofs and Puzzles

A geometric proof involves writing reasoned logical explanations that use definitions, postulates, previously proved theorems to arrive at a conclusion. Proofs are like puzzles because we piece together information that helps us prove what we are learning in geometry.

We have three ways that we can write proofs:

1. **Flowchart Proof**
   - is a concept map that shows the statements and reasons needed for a proof in a structure that helps to indicate the logical order.

2. **Paragraph Proof**
   - is a proof written in the form of a paragraph.

3. **2-Column Proof**
   - geometric proof consists of a list of statements, and the reasons that we know those statements are true.

Engage: Example of Flowchart Proof

Prove $\overline{ED} \cong \overline{FD}$
Engage: Graphic Organizer/Notes

Engage: Example of a Paragraph Proof

Prove $\overline{ED} \cong \overline{FD}$

It is given that $\angle EGD$ and $\angle FGD$ are right angles and $\overline{ED} \cong \overline{FD}$. According the definition of right triangles, $\triangle EGD$ and $\triangle FGD$ are right triangles. $\overline{GD}$ is also congruent to $\overline{GD}$ by Reflexive Property. $\triangle EGD \cong \triangle FGD$ are congruent by the Hypotenuse-Leg Theorem because the corresponding hypotenuse and leg of one triangle is congruent to the other. Lastly, $\overline{ED} \cong \overline{ED}$ by CPCTC (Congruent Parts of Congruent Triangles are Congruent).

Explore: Angle Puzzles

Use the pictures below and fill in the missing pieces or angle measurements.

1.
## Explore: Example of Vertical Angles Proof

In this diagram, line \( m \) intersects line \( n \).

Write a two-column proof to show that vertical angles \( \angle 1 \) and \( \angle 3 \) are congruent.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line ( m ) intersects line ( n )</td>
<td>Given</td>
</tr>
<tr>
<td>( \angle 1 ) and ( \angle 2 ) form a linear pair, ( \angle 2 ) and ( \angle 3 ) form a linear pair</td>
<td>Definition of a linear pair</td>
</tr>
<tr>
<td>( m\angle 1 + m\angle 2 = 180^\circ )</td>
<td>Angles that form a linear pair have measures that sum to 180°</td>
</tr>
<tr>
<td>( m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 )</td>
<td>Substitution</td>
</tr>
<tr>
<td>( m\angle 1 = m\angle 3 )</td>
<td>Subtraction Property of Equality</td>
</tr>
<tr>
<td>( \angle 1 \cong \angle 3 )</td>
<td>Definition of congruent angles</td>
</tr>
</tbody>
</table>
Apply: Guided Practice Problem

Give the reason for Step 7 (See Below)

In this diagram, \( \overline{CD} \) is the perpendicular bisector of \( \overline{AB} \). The two-column proof shows that \( \overline{AC} \) is congruent to \( \overline{BC} \).

**Step 1:** Mark the picture according to what is given in the problem. (Use the sketch tool to mark your picture).

**Step 2:** List out the given information.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{CD} ) is the perpendicular bisector of ( \overline{AB} )</td>
<td>Given</td>
</tr>
</tbody>
</table>

**Step 3:** Give the statement and reasons for a Perpendicular Bisector.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{AB} \cong \overline{BD} )</td>
<td>Definition of a Bisector</td>
</tr>
</tbody>
</table>

Georgia Department of Education
April 2021
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**Step 4:** Give a statement and reason for Reflexive Property.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{CD} \cong \overline{CD}$</td>
<td>Reflexive Property</td>
</tr>
</tbody>
</table>

**Step 5:** Give a statement and reason for Perpendicular Lines.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle ADC$ and $\angle BDC$</td>
<td>Definition of Perpendicular Lines.</td>
</tr>
</tbody>
</table>

**Step 6:** Give a statement and reason about all right angles being congruent.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle ADC$ and $\angle BDC$</td>
<td>All right angles are congruent</td>
</tr>
</tbody>
</table>

**Step 7:** Give a statement and reason about congruent triangles. List out your answer below:

Hint: Are the triangles congruent by SSS, SAS, ASA, AAS, or HL?

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ADC \cong \triangle BDC$</td>
<td></td>
</tr>
</tbody>
</table>
### Step 8: Give a statement and reasons for Congruent Parts of Congruent Triangles are Congruent (CPCTC)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ADC \cong \triangle BDC$</td>
<td>____________</td>
</tr>
<tr>
<td>$AC = BC$</td>
<td>CPCTC</td>
</tr>
</tbody>
</table>
Give the Reasons for Steps 1, 4, 5, and 6

Apply: Proof of the Sum of the Angles in a Triangle

Write a two-column proof that proves the sum of the angles in a triangle is 180°.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{XY} \parallel \overline{AC} ) and ( \overline{AB} ) is a transversal.</td>
<td>1. ______________</td>
</tr>
<tr>
<td>( \angle A ) and ( \angle ABX ) are alternate interior angles.</td>
<td>2. Definition of alternate interior angles.</td>
</tr>
<tr>
<td>( \angle A \cong \angle ABX )</td>
<td>3. Alternate Interior Angles Theorem</td>
</tr>
<tr>
<td>( \angle C ) and ( \angle CBY ) are alternate interior angles.</td>
<td>4. ______________</td>
</tr>
<tr>
<td>( \angle C \cong \angle CBY )</td>
<td>5. ______________</td>
</tr>
<tr>
<td>( m\angle ABX + m\angle ABC + m\angle CBY = 180° )</td>
<td>6. ______________</td>
</tr>
<tr>
<td>( m\angle A + m\angle ABC + m\angle C = 180° ).</td>
<td>Substitution</td>
</tr>
</tbody>
</table>

In this diagram, \( \overline{XY} \) is parallel to \( \overline{AC} \), and point \( B \) lies on \( \overline{XY} \).
Apply: Put the reasons in the correct order so that they line up with the statements.

*Each problem is on 2 different pages

1. Parallelogram Proof

In this diagram, \(ABCD\) is a parallelogram and \(BD\) is a diagonal.

Write a two-column proof to show that \(AB\) and \(CD\) are congruent.
Put the Reasons in the correct order.

<table>
<thead>
<tr>
<th>Step</th>
<th>Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ABCD is a parallelogram.</td>
</tr>
<tr>
<td>2</td>
<td>Line segment BD is a diagonal.</td>
</tr>
<tr>
<td>3</td>
<td>Line segment AB is parallel to DC. Line segment AD is parallel to BC</td>
</tr>
<tr>
<td>4</td>
<td>( \triangle ABD \cong \triangle CDB ) and ( \angle DBC \cong \angle BDA )</td>
</tr>
<tr>
<td>5</td>
<td>Line segment BD ( \parallel ) BD</td>
</tr>
<tr>
<td>6</td>
<td>( \triangle ADB \cong \triangle CBD )</td>
</tr>
<tr>
<td>7</td>
<td>Line segment AB ( \parallel ) CD</td>
</tr>
</tbody>
</table>

**REASONS**

- Alternate interior angles are congruent.
- Definition of parallelogram
- ASA
- CPCTC
- Given
- Reflexive Property of Congruence
- Given

**PUT IN CORRECT ORDER (HINT: MATCH REASONS TO THE STATEMENTS)**
2. CPCTC Proof

STU is an isosceles triangle. Line segment DM is an angle bisector.

Proof that \( \angle E \cong \angle F \)

<table>
<thead>
<tr>
<th>Step</th>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Line segment DE = DF</td>
<td>SAS</td>
</tr>
<tr>
<td>2</td>
<td>Line segment DM is an angle bisector</td>
<td>Given</td>
</tr>
<tr>
<td>3</td>
<td>( \angle EDM = \angle FDM )</td>
<td>Given</td>
</tr>
<tr>
<td>4</td>
<td>Line segment DM \cong DM</td>
<td>CPCTC</td>
</tr>
<tr>
<td>5</td>
<td>( \triangle DEM \cong \triangle DFM )</td>
<td>Reflexive Property of Congruence</td>
</tr>
<tr>
<td>6</td>
<td>( \angle E \cong \angle F )</td>
<td>Definition of Angle Bisector</td>
</tr>
</tbody>
</table>
3. Parallelogram Proof

Put the Reasons in the correct order.

<table>
<thead>
<tr>
<th>Step</th>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AD and BC bisect each other at point F.</td>
<td>SAS</td>
</tr>
<tr>
<td>2</td>
<td>AF = DF, BF = CF</td>
<td>Given</td>
</tr>
<tr>
<td>3</td>
<td>(\angle AFB = \angle DFB) and (\angle AFC = \angle BFD)</td>
<td>Vertical Angles Theorem</td>
</tr>
<tr>
<td>4</td>
<td>(\triangle AFB = \triangle DFB) and (\triangle AFC = \triangle BFD)</td>
<td>Definition of a Parallelogram</td>
</tr>
<tr>
<td>5</td>
<td>(\angle CAD = \angle BDA) and (\angle CDA = \angle BAD)</td>
<td>CPCTC</td>
</tr>
<tr>
<td>6</td>
<td>AB // CD and BD // AC</td>
<td>Converse of Alternative Interior Angles</td>
</tr>
<tr>
<td>7</td>
<td>ABCD is a Parallelogram</td>
<td>Definition of Segment Bisector</td>
</tr>
</tbody>
</table>
4. Mid-Segment Theorem Proof

Given: \( \overline{AD} \cong \overline{BD} \) and \( \overline{AE} \cong \overline{CE} \)
Prove: \( \overline{DE} \parallel \overline{BC} \)

Put the Reasons in the correct order.

<table>
<thead>
<tr>
<th>Step</th>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \overline{AD} = \overline{BD} ) and ( \overline{AE} = \overline{CE} )</td>
<td>Midsegment Theorem</td>
</tr>
<tr>
<td>2</td>
<td>Point D is the midpoint of AB</td>
<td>Definition of a Midpoint.</td>
</tr>
<tr>
<td>3</td>
<td>Point E is the midpoint of AC</td>
<td>Given</td>
</tr>
<tr>
<td>4</td>
<td>DE is the mid-segment of ( \triangle ABC )</td>
<td>Definition of a Midpoint.</td>
</tr>
<tr>
<td>5</td>
<td>DE \parallel BC</td>
<td>Definition of a midsegment</td>
</tr>
</tbody>
</table>
Create a Flowchart Proof

Click on this link to access an interactive Google Slides Activity: Click Here
Now Create a Paragraph Proof

\( \triangle STU \) is an isosceles triangle. \( \overline{DN} \) is an angle bisector. 
Prove that \( \angle E \cong \angle F \).
Δ STU is an isosceles triangle. \( \overline{DM} \) is an angle bisector. Prove that \( \angle E \cong \angle F \).