Georgia Mathematics Strategies Toolkit to Address Learner Variability for Grades K - 5

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Introduction

“If the goal in mathematics teaching and learning is to support student success with mathematical proficiency, then we must be explicit about using instructional routines that focus on student engagement in activities that support reasoning and sense making, communication with and about mathematical ideas, making meaningful connections, building procedural fluency from conceptual understanding…”

-Thinking about Instructional Routines in Mathematics Teaching and Learning

Within this toolkit, educators will find observations of student behavior for each of the 13 identified areas for addressing learner variability. Aligned to each observation of student behavior, are evidence-based, research-based strategies intended to strengthen students’ ability in mathematics. Support resources are provided to assist educators with implementing the strategies.

- Behavior
- Cognitive Processing
  - Attention
  - Conceptual
  - Memory
  - Reasoning
- Executive Functioning
- Instructional Climate and Student Mindsets
- Language Processing
- Language Proficiency
- Mathematics Calculation
- Other Exceptionalities
- Problem-Solving
- Visual-Spatial Processing

Looking for more evidence-based, researched based practices for mathematics? Please visit gadoe.org/mathematics.
Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning. (EMTP 1)
2. Implement tasks that promote reasoning and problem solving. (EMTP 2)
3. Use and connect mathematical representations. (EMTP 3)
4. Facilitate meaningful mathematical discourse. (EMTP 4)
5. Pose purposeful questions. (EMTP 5)
6. Build procedural fluency from conceptual understanding. (EMTP 6)
7. Support efforts of learning in mathematics. (EMTP 7)
8. Elicit and use evidence of student thinking. (EMTP 8)

Mathematical Practices

1. Make sense of problems and persevere in solving them. (MP.1)
2. Reason abstractly and quantitatively. (MP.2)
3. Construct viable arguments and critique the reasoning of others. (MP.3)
4. Model with mathematics. (MP.4)
5. Use appropriate tools strategically. (MP.5)
6. Attend to precision. (MP.6)
7. Look for and make use of structure. (MP.7)
8. Look for and express regularity in repeated reasoning. (MP.8)
**Behavior**

**Mathematics Connection:** When students develop productive mathematical behavior, they are confident in their ability to attain proficiency and outstanding performance on an assignment. They are not afraid to ask questions or seek assistance when needed, and they are confident and display limited frustration when grappling with challenging problems.

**Student Learning Expectations:** Students should have opportunities to develop a growth mindset and transform to show a mathematical mindset by gaining confidence, taking instructional risks, and asking questions for clarity on challenging activities to achieve at their optimal performance.

<table>
<thead>
<tr>
<th>Behavior</th>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
</table>
| Observations reveal inattentiveness, and frustration, such as: staring into space, whining/crying saying, “I can't do this.” | The teacher will establish clear mathematical goals for learning and use the goals to guide instruction. (EMTP 1)  
- The teacher will set goals and expectations for learning using learning progressions.  
- The teacher will explicitly teach strategic tools/actions for problem-solving.  
The teacher will support efforts of learning in mathematics. (EMTP 7)  
- The teacher will purposefully build on students’ prior knowledge.  
- The teacher will use evidence-based instructional strategies. | The student will make sense of problems and persevere in solving them. (MP.1)  
- The student will set clear goals aligned to desired learning objectives.  
- The student will use strategic tools/actions when solving routine and non-routine problems. | Guide for Effective Mathematics Instruction  
These mathematics toolkits provide strategies to identify gaps in learning and evidence-based strategies to address the gaps.  
Self-regulating strategies  
4 tips on self-regulating strategies. |
| Observations reveal disruption of the learning environment, such as students talking to other students during instruction. | The teacher will implement tasks that promote reasoning and problem solving. (EMTP 2)  
- The teacher will use a variety of instructional strategies that result in active student participation.  
- The teacher will motivate students’ learning of mathematics through opportunities | The student will reason abstractly and quantitatively. (MP.2)  
- The student will take ownership for making sense of tasks by referencing and making connections with their prior understanding and ideas.  
- The student will persevere in exploring and reasoning through tasks. | High Engagement Strategies  
This Resource is an article on strategies for increasing Mathematical engagement.  
3-Act Lessons  
A Three-Act task or lesson is a whole-group mathematics task consisting of three distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion |
for inquiry, exploration and solving problems.

The teacher will facilitate meaningful mathematical discourse. (EMTP 4)
- The teacher will create opportunities for collaboration among students.

The student constructs viable arguments and critique the reasoning of others. (MP.3)
- The student will engage in the use of turn and talks, think-pair-share, gallery walks and math stations.

and solution revealing Act Three.

Cognitive Processing: Attention, Conceptual, Memory, Reasoning

Mathematics Connection: Cognitive processes include developmental processes such as attention, memory, reasoning, and conceptual processes; when used by students every day as a support strategy, these strategies will allow students to develop profound connections of the concepts, be able to recognize relationships, and perform cognitive procedures with situational-based problems. Once students make the connection, they are able to transfer that new acquired thinking to problem solving.

Student Learning Expectations: Students should have opportunities to use cognitive process strategies as a learning support tool to bridge the prior ideas with the new learning concepts. Students are now fully equipped with the ability to perform the procedures to arrive at a solution.

Cognitive Processing: Attention

Mathematics Connection: Increasing complexity of mathematics content and tasks require extended attention spans. Maintaining attention can be challenging if a student is uninterested or the task seems too challenging.

Student Learning Expectations: Students should have opportunities to focus on a portion of any given information to concentrate on the relevant content, while using cognitive attention processes to remain focused on the pertinent information.

<table>
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<tr>
<th>Attention</th>
<th>Teacher Actions</th>
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</table>
| Observations reveal difficulty staying engaged during mathematics discourse. | The teacher will facilitate meaningful mathematical discourse. (EMTP 4) | The student construct viable arguments and critique the reasoning of others. (MP.3) | Effective Instructional Practices Guide
Instructional support guide for implementing learning experiences that encourage mathematical discourse. |
| Observations reveal difficulty recalling prior mathematics skills or concepts previously taught. | The teacher will pose purposeful questions to assess student prior knowledge and elicit student | The student will use math models to build conceptual understanding of the | Prior Knowledge Warm-Up Activities
In mathematics lessons, most warm-up activities |
thinking to address concepts needing review. (EMTP 5 & 8)

previous skills and apply them to current content. (MP.4)

that activate prior knowledge are generated either by a mathematical task or a discussion prompt you provide for the students as class begins.

| Observations reveal difficulty sustaining engagement in group work. | The teacher will facilitate mathematical discourse to build shared understanding of mathematical concepts through implementing group roles in math tasks. (EMTP 4) | Students will listen to groupmates, explain and justify their thinking, and ask each other questions about their solutions. (MP.3) | Setting Up Effective Group Work Article
This article will aid the teacher by providing options to role setup in group work. |

**Cognitive Processing: Conceptual**

**Mathematics Connection:** Conceptual understanding involves meaningfully learned and well-integrated knowledge about mathematics. Students should gain many logical connections between specific concepts and ideas.

**Student Learning Expectations:** Students should have opportunities to use conceptual processing to help make sense of real-life, mathematical problems to gain a deeper understanding of the integrated knowledge and the connections of the big idea concepts.

### Conceptual

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</table>
| Observations reveal lack of self-checking or metacognitive skills, for example, in basic facts. | The teacher will pose purposeful questions to aid the student in metacognitive skills. (EMTP 5) | The student will restate the meaning of the problem, make choices about the solution path and alter as needed - assessing the reasonableness along the way. (MP.1, 2 & 7) | GA Frameworks Video: MGSE1.OA.1
In this video, notice the questioning technique of the teacher and how it aids students’ metacognitive skills. |
| Observations reveal difficulty applying previously learned concepts to novel situations, for example: identifying plane figures or place value with operations. | The teacher will pose purposeful questions to assess student prior knowledge and to guide next steps in experiences to provide. (EMTP 5) | The student will recognize patterns in numbers, diagrams, and graphs, using those patterns previously learned to make generalizations about novel situations. (MP.7) | Contemplate then Calculate
An instructional routine designed to shift attention away from mindless calculations and toward necessary structural interpretations of mathematics. GA Frameworks Video: MGSE.K.G.6
Video showing questioning techniques of the teacher and how it helps students connect previously learned knowledge. |
Cognitive Processing: Memory

**Mathematics Connection:** Using rote memory to recall facts or remember the steps of an algorithm does not yield long-term learning. Students should have opportunities to use conceptual learning strategies that will lead to committing the basic computational facts to memory.

**Student Learning Expectations:** Students should have opportunities to use knowledge from their memories to perform calculations and procedures, identify geometric figures, and demonstrate basic graphing skills by using visual-spatial and numerical representations to make sense of real-life, mathematical problems to help with sustaining long-term memory.

### Memory

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<tr>
<td>Observations reveal difficulty recalling key vocabulary terms consistently, such as: number names, the difference between area and perimeter, etc.</td>
<td>The teacher will facilitate meaningful discourse that allows students to repeat accurate math vocabulary while engaging in rich tasks. (EMTP 4)</td>
<td>The student will demonstrate precise communication of mathematical ideas using clear academic-language and accurate vocabulary. (MP.6)</td>
<td>Counting Cup Lesson&lt;br&gt;This video shows how teachers explicitly use and reinforce vocabulary terms with manipulatives. GA Frameworks Task on Perimeter and Area&lt;br&gt;This GA frameworks task allows students to demonstrate their knowledge and distinguish between concepts.</td>
</tr>
<tr>
<td>Observations reveal difficulty recalling prior mathematics skills or concepts previously taught.</td>
<td>The teacher will pose purposeful questions to assess student prior knowledge and elicit student thinking to address concepts needing review. (EMTP 5)</td>
<td>The student will use math models to build conceptual understanding of the previous skills and apply them to current content. (MP.4)</td>
<td>GA Frameworks Video: MGSEK.CC.4&lt;br&gt;This video is part of the GA Frameworks video series. Here, the teacher demonstrates questioning skills that connect previous learning to new knowledge. Addition and Subtraction Progression Video&lt;br&gt;This video provides the teacher with knowledge of the sequencing of skills.</td>
</tr>
</tbody>
</table>
Cognitive Processing: Reasoning

Mathematics Connection: Mathematical reasoning enables students to make use of the concepts and skills in the lesson by gaining access to understanding. Students evaluate the situations, select problem-solving strategies, draw conclusions, develop and describe solutions and situational context. When students apply mathematical reasoning, they can reflect on the contextual, mathematical problems.

Student Learning Expectations: Students should have opportunities to critique and explain their thinking & to make sense of the problem and the reasoning of others as they engage in contextual, mathematical situations.

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<thead>
<tr>
<th>Reasoning</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
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<tbody>
<tr>
<td>Observations reveal difficulty in distinguishing between concepts by attribute, for example circle, rectangle, and triangle.</td>
<td>The teacher will pose purposeful questions to assess prior knowledge and elicit student thinking to aid in distinguishing between concepts. (EMTP 5 &amp; 8)</td>
<td>The student will reason abstractly using symbols and drawings to represent their ideas, for instance: describing shapes by attributes and classifying them into categories. (MP.2)</td>
<td>Sophisticated Shapes&lt;br&gt;This is a Kindergarten unit plan that allows for students to demonstrate their abstract thinking.</td>
</tr>
<tr>
<td>Observations reveal difficulty explaining why computational strategies work, for example: those using place value and the properties of addition and subtraction.</td>
<td>The teacher will pose purposeful questions to assess prior knowledge and elicit student thinking to address concepts needing review. (EMTP 5) The teacher will use and connect mathematical representations. (EMTP 3) • The teacher will incorporate instructional time for students to use, discuss, and make connections among representations. • The teacher will make explicit connections among mathematical concepts and representations.</td>
<td>The student will make sense of the strategy, make use of the strategy structure, and attend to precision in their explanation. (MP.1, 2, 6 &amp; 7)</td>
<td>GA Frameworks Video: MGSEKG6&lt;br&gt;In this video the teacher demonstrates questioning skills that allow students to distinguish between plane shape vocabulary.</td>
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Connecting Representations
An instructional routine that positions students to think structurally as they connect two representations by articulating the underlying mathematics.

Notice and Wonder
When students become active doers of mathematics, the greatest gains of their mathematical thinking can be realized. The process of sense making begins when we create classrooms full of curious students’ thoughts and ideas.
Executive Functioning

**Mathematics Connection:** Executive functioning is a set of cognitive mental competencies that students use to organize information and put it into use while working on mathematical problems, visualize problems from multiple angles, come up with different ways to solve them, and control attention and behavior to manage emotions when encountering the efforts of learning.

**Student Learning Expectations:** Students should have opportunities to make sense of problems and persevere in solving them while making use of structure and reasoning about the repeated use of structure. Students will also be expected to model with mathematics.

<table>
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<tr>
<th>Executive Functioning</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</table>
| Observations reveal difficulty completing multi-step word problems and tasks. | The teacher will implement tasks that promote reasoning and problem solving. (EMTP 2)  
  - The teacher will select tasks that provide multiple entry points.  
  - The teacher will encourage the use of varied approaches and strategies to make sense of and solve problems.  
  - The teacher will provide opportunities for students to use their own reasoning strategies and methods for solving problems. | The student will make sense of the problem and persevere in solving them as they reason about the quantities in the problem. (MP.1)  
  - The student will use problem solving tools and representations, as needed, such as graphic organizers.  
  - The students will make connections with their prior understanding and ideas to make sense of problems. | 3-Act Lessons  
A Three-Act task or lesson is a whole-group mathematics task consisting of three distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. |

| Observations reveal difficulty making connections and understanding the relationship between symbols and quantities. | The teacher will use and connect mathematical representation. (EMTP 3)  
  - The teacher will use relatable items/interest to make connections and for representation.  
  The teacher will implement tasks that promote reasoning and problem solving. (EMTP 2) | The student will use manipulatives (base ten blocks) as mathematical symbols of representation when solving contextual mathematical problems. (MP.1, 2, & 4) | Making-Mathematical-Connections  
This resource is an article on mathematical connections.  
Tour of Mathematical Connections  
This short tour shows the connected nature of math highlighting number sense, geometry, and algebra. |
Instructional Climate and Student Mindsets

Mathematics Connection: A student's mindset and disposition can influence how they understand, connect, and recognize relationships in mathematics. A connection between mathematical mindsets and learning must be addressed if mathematics is going to be accessible to all students. Students’ mathematical understanding is developed over time; therefore, it is important to recognize how students’ understanding of their abilities and the instructional climate can affect mathematics conceptual learning.

Student Learning Expectations: Students should have opportunities to see themselves as numerate contributors to the world around them and beyond and think deeply about mathematics as they prepare to enter the future workforce with the critical thinking and reasoning skills necessary for success in both the local and global workforces.

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### Instructional Climate and Student Mindsets

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<tr>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
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<tbody>
<tr>
<td>Observations reveal student focus and engagement affect participation and performance during mathematics lessons.</td>
<td>The teacher will implement tasks and provide opportunities that bridge the mathematics classroom with the community.</td>
<td>The student will discuss the meaning of problems with classmates, explore other ways to solve problems, try out potential solutions, check for reasonableness and make changes as necessary (MP.1 &amp; 3)</td>
<td>5 Teaching Strategies to build math confidence</td>
</tr>
<tr>
<td>The teacher will create a positive learning environment that encourages students to see the connections within their everyday lives. Students are celebrated for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. (EMTP 2 &amp; 7)</td>
<td>The teacher will use a variety of strategies to build community &amp; grasp a deeper understanding of students’ mathematical interest and perceptions of their mathematical ability. (EMTP 8)</td>
<td>The student will explain and justify their thinking using words, objects, and drawings. (MP.2)</td>
<td></td>
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<tr>
<td>The student will discuss the meaning of problems with classmates, explore other ways to solve problems, try out potential solutions, check for reasonableness and make changes as necessary (MP.1 &amp; 3)</td>
<td>The student will use mathematical models to solve problems. (MP.4)</td>
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</table>
Observations reveal misconceptions of mathematical understanding and conceptualization among varied learners.

The teacher will facilitate discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. (EMTP 4)
- The teacher will use peer tutoring and collaboration as an instructional practice.

The teacher will provide and support students with high quality and challenging learning opportunities. (EMTP 7)

The student will discuss the meaning of problems with classmates, explore other ways to solve problems, try out potential solutions, check for reasonableness and make changes, as necessary. (MP.1, 3 & 7)

The student will explain and justify their thinking using words, objects, and drawings. (MP.2)

The student will use mathematical models to solve problems. (MP.4)

7 Real-World Math Strategies
How do you use the real world to teach math?
Language Processing

**Mathematics Connection:** Language processing is essential to mathematical discourse. Mathematical discourse is about the exchange of mathematical ideas, including ways of representing, thinking, talking, agreeing, and disagreeing. Students who are satisfactorily processing language can explain their thinking and articulate the relationships they see in mathematics. In addition, students can ask questions to help clarify an argument of others and explain what makes strong reasoning.

**Student Learning Expectations:** Students should have opportunities to express mathematical ideas with precision and clarity.

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<tr>
<th>Language Processing</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
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</table>
| Observations reveal brief and/or incomplete oral or written explanations of reasoning or solution strategies. | The teacher will facilitate sustained & reciprocal discourse through sentence frames and challenging students to create their own math problems. (EMTP 4) | The student will make sense of words and problems while using clear language and accurate math vocabulary. (MP.1 & 6) | Strategies for Increasing Language Production  
This article aids the teacher by providing tips to aid in writing and speaking.  
Recommendation 2 Integrate Oral and Written English Language Instruction into Content-Area Teaching  
This practice guide provides four recommendations that address what works for English learners during reading and content area instruction. |
| Observations reveal difficulty in producing language to ask questions that effectively express what student needs to know. | The teacher will facilitate sustained & reciprocal discourse through providing the opportunity for creative thought and mathematical reasoning through journaling. (EMTP 4) | The student will use words that express appropriate meaning, thus becoming more able to articulate questions and their own understanding. (MP.6) | Mathematics Language Routines 5: Co-Craft Questions  
Through this routine, students can use conversation skills to generate, choose, and improve questions and situations.  
Math Journal Examples  
This resource provides many journal prompts and questions that elicit conceptual thinking and creativity. |
Language Proficiency

Mathematics Connection: Language proficiency is essential to mathematical literacy and comprehension. Language proficiency involves students’ ability to understand and use the language of mathematics to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing.

Student Learning Expectations: Students should have opportunities to precisely use mathematical language to explain their thinking and reasoning in mathematics.

<table>
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<tr>
<th>Language Proficiency</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tr>
<td><strong>Observations</strong></td>
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<tr>
<td>Observations reveal difficulty</td>
<td>The teacher will facilitate meaningful discourse. (EMTP 4) • The teacher will use purposeful techniques to improve comprehension of mathematical words, phrases, and sentences.</td>
<td>The student will make sense of problems through repetition and using provided scaffolds, persevering even when faced with difficulty. (MP.1)</td>
<td>Proficiency Level Descriptors PLDs describe typical ways multilingual learners might develop across levels of English language proficiency as they move toward meeting Language Expectations.</td>
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<tr>
<td>Observations reveal difficulty in demonstrating comprehension of mathematical text in the context of a word problem.</td>
<td>(EMTP 4)</td>
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<tr>
<td>Observations reveal difficulty in using language to communicate a mathematical idea.</td>
<td>The teacher will facilitate meaningful discourse through providing the opportunity for creative thought and mathematical reasoning through journaling. (EMTP 4) • The teacher will explicit teach causal connectors to express reasoning. • The teacher will provide multiple opportunities for structured peer interactions or conversations (pairs or triads) to negotiate meaning using charts, graphic organizers, a word bank and/or sentence frames.</td>
<td>The student will explain and justifying their thinking using words, objects, and drawings. (MP.3) • The student will use language choices to reflect on completed and on-going process. • The student will record mathematical reasoning in English through journaling or structured peer interactions.</td>
<td>Mathematical Language Routines to Optimize Student Output 1: Stronger and Clearer Each Time This routine also provides a purpose for student conversation using a discussion-worthy and iteration-worthy prompt.</td>
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<tr>
<td><strong>Support Resources</strong></td>
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<td><strong>ProficiencyLevel Descriptors</strong></td>
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<tr>
<td><strong>Strategies for Increasing Language Production</strong></td>
<td>This article aids the teacher by providing tips to aid in writing and speaking.</td>
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<tr>
<td><strong>Sentence Stems to Encourage Math Talks</strong></td>
<td>This website provides “25 Sentence Stems to Encourage Math Talk” along with a guide to how to use the sentence stems in a math class.</td>
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Mathematics Calculation

Mathematics Connection: Mathematical calculation involves using mathematical or logical reasoning to solve a problem. When demonstrating math computation, students understand and use mathematical language, learn processes and procedures for solving math problems, and reflect on solutions to problems to determine if they make sense.

Student Learning Expectations: Students should have opportunities to show growth development in number sense to perform math calculations fluently by using the connections of understanding between visual-spatial and numerical representations.

<table>
<thead>
<tr>
<th>Mathematics Calculations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
<tr>
<td>Observations reveal difficulty connecting mathematical ideas and concepts across representations and effectively apply math concepts/skills. For example, using the four operations for computation of multi-digit numbers.</td>
<td>The teacher will provide tasks that promote mathematical reasoning and allow varied solution strategies. (EMTP 2) The teacher will provide learning experiences that allow students to progress from the concrete level to the representational level to the abstract level. (EMTP 3)</td>
<td>The student will explain the connection between different representations. (MP.4) • The student will demonstrate problem solving skills in multiple ways including the use of objects, acting out, pictorial representations, making a chart or list, creating equations, etc.</td>
<td>Concrete, Representational, Abstract Chart This resource describes each stage of the strategy as well as suggested questions to use during implementation.</td>
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<tr>
<td>Observations reveal inability to use estimation strategies to assess the reasonableness of a solution.</td>
<td>The teacher will facilitate discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. (EMTP 4) The teacher will provide students with opportunities to use their own reasoning strategies and methods for solving problems. (EMTP 6)</td>
<td>The student will detect possible errors by strategically using estimation and other mathematical knowledge. (MP.5)</td>
<td>Scaffolding Task: By the Riverside Students explore problem-based lessons that consist of three main parts: before, during and after. Constructing Task: Mental Mathematics Students will practice estimation and explaining strategies.</td>
</tr>
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</table>
Other Exceptionalities

**Mathematics Connection:** Instruction must incorporate strategies and supports providing access to the content within the general education curriculum for mathematics, rich learning tasks that allow for the implementation of the Mathematical Practices and the building of proficiency in mathematics for all learners through sustained and deep engagement in practices that include problem solving, reasoning, and critiquing.

**Student Learning Expectations:** Students should have opportunities to meet high academic standards and demonstrate the level of mathematical reasoning needed to fully develop their conceptual understanding and procedural fluency.

<table>
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<tr>
<th>Other Exceptionalities</th>
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</table>
| Observations reveal difficulty with fine motor skills including the use of manipulatives. | The teacher will support efforts in learning mathematics. (EMTP 7)  
- The teacher will provide learning experiences with a variety of digital tools.  
The teacher will use and connect mathematical representations. (EMTP 3)  
- The teacher will introduce forms of representations that can be useful to students. | The student will model with mathematics. (MP.4)  
The student will use appropriate tools strategically. (MP.5)  
- The student will use a variety of tools which may include digital resources.  
The student will attend to precision. (MP.6)  
- The student will engage in peer collaboration. | **Virtual Math Manipulatives**  
This resource is a link to multiple virtual manipulatives.  
**Desmos**  
A free suite of math software tools, including the renowned Desmos Graphing Calculator and Scientific Calculator, are used annually by over 40 million teachers and students around the world. |
| Observations reveals difficulty with verbal communication while engaging/collaborating in math class. | The teacher will facilitate meaningful mathematical discourse. (EMTP 4)  
- The teacher will select and sequence student approaches and solution strategies for whole-class analysis and discussion.  
- The teacher will revoice student ideas to demonstrate mathematical language use by restating a statement. | The student will construct arguments using concrete referents such as objects, drawings, diagrams, and actions. (MP.2 & 3)  
The student will listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (MP.3) | **Mathematics Language Routine 8: Discussion Supports**  
To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.  
**HLP 18: Use Strategies to Promote Active Student Engagement**  
Teachers use a variety of instructional strategies that result in active student responding. |
**Problem-Solving**

**Mathematics Connection:** Problem-solving is a cognitive process that involves students learning mathematics through contextual, mathematical problems and models. The contexts and models allow students to build meaning for the concepts so they can fully understand abstract concepts.

**Student Learning Expectations:** Students should have opportunities to use mathematical reasoning to make sense of the content, extract information and explore new concepts as they persevere in solving contextual, mathematical problems.

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<tbody>
<tr>
<td>Observations reveal an inability to process and solve word problems.</td>
<td>The teacher will intentionally select tasks that provide multiple entry points using varied tools and representations. (EMTP 2) - The teacher will provide scaffolds to support sense making of problems by deconstructing the process of reading mathematical situations.</td>
<td>The student will explain the meaning of a problem and look for ways to discuss how they solved it. (MP.1, 2) - The student will create a representation of a problem while attending to the meanings of the quantities.</td>
<td>3 Reads Instructional Routine Designed to develop students’ ability to make sense of problems by deconstructing the process of reading mathematical situations. Constructing Task: Addition and Subtraction Word Problems This task provides an example of how you recreate this constructing task to use throughout addition and subtraction units.</td>
</tr>
<tr>
<td>Observations reveal difficulty identifying quantities and relationships in a problem statement.</td>
<td>The teacher will intentionally choose tasks that provide students with scaffolding and tools that promote mathematical reasoning and problem solving. (EMTP 1) The teacher will use purposeful questions to assess and advance students’ reasoning and sense making. (EMTP 5)</td>
<td>The student will visualize structures and patterns in problems. (MP.4) The student will justify their thinking using mathematical languages. (MP.3 &amp; 6)</td>
<td>3 Read Resources Designed to develop students’ ability to make sense of problems by deconstructing the process of reading mathematical situations. Capturing Quantities An instructional routine designed to focus students’ attention on important quantities and relationships in problem situations.</td>
</tr>
<tr>
<td>Observations reveal difficulty understanding written or verbal directions or explanations.</td>
<td>The teacher will facilitate meaningful mathematical discourse. (EMTP 6)</td>
<td>The student will attend to precision. (MP.6)</td>
<td>HLP 12: Systemically design instruction toward a specific learning goal. High-leverage practice.</td>
</tr>
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Visual-Spatial Processing

**Mathematics Connection:** Students create and use visual representations to solve problems and to explore and communicate mathematical concepts and ideas in the mathematical domains. Visual-spatial processing affects students’ ability to solve multi-step problems, analyze certain representations, interpret and manipulate geometric configurations and appreciate changes in objects as they are moved in space.

**Student Learning Expectations:** Students should have opportunities to experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, and creating equations. Students will be expected to connect the different representations and explain the connections.

### Visual-Spatial Processing

<table>
<thead>
<tr>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
<tr>
<td>Observations reveal inability to connect the change in size or orientation of a shape to its name.</td>
<td>The teacher will intentionally select tasks that allow students to practice applying and strengthening visual-spatial skills. (EMTP 2) The teacher will observe and effectively responds to students’ thinking. (EMTP 8)</td>
<td>The student will create and use visual representations to solve problems and to explore and communicate mathematical concepts and ideas in the mathematical domains. (MP.3,4,6)</td>
<td>Avoiding Math Taboos: Effective Math Strategies for Visual-Spatial Learners Articles explains ways in which visual-spatial learners process information. Scaffolding Task: Listen and Do! Students practice spatial relationships.</td>
</tr>
<tr>
<td>Observations reveal difficulty perceiving numbers as individual units and performing operations with multiple digits.</td>
<td>The teacher will use and connect mathematical representations. (EMTP 3) ● The teacher will provide copies of notes, graph paper to align numbers, and utilize color coding to organize information to connect mathematical representations.</td>
<td>The student will discern patterns or structures. (MP.7)</td>
<td>Positive Effects of Manipulatives in the Classroom This website describes how manipulatives are useful for instruction. 4:9 Recalling the number of 10s within decades that add to 100 Numeracy Intervention Task and Activity skill.</td>
</tr>
<tr>
<td>Observations reveal difficulty analyzing problems and represent problem solving in multiple ways.</td>
<td>The teacher will present students with real-world and mathematical problems. (EMTP 2) ● The teacher will intentionally select tasks that allow students to strengthen and apply visual-spatial mathematical skills.</td>
<td>The student will use background/prior knowledge to make sense of problems and persevere in solving them by exploring and investigating skills and tools needed to solve problems. (MP.1, 2 &amp; 7)</td>
<td>Routines for Reasoning Instructional Routines are specific and repeatable designs for learning that support both the teacher and students in the classroom.</td>
</tr>
</tbody>
</table>
Descriptions

Behavior
Behavior is a form of communication. Every behavior has a function. A growth mindset in mathematics can be demonstrated through the students’ confidence and perseverance in their thinking and executing, asking questions for clarity, and grappling with context-based problems. These students are willing to tackle challenging problems and take learning risks.

Cognitive Processing
Cognitive processes allow students to make connections between prior concepts and new ideas. When students are aware and use at least one of the four mathematics cognitive processes, they will be capable of transferring cognitive skills into their learning.

Attention
Attention is a cognitive process that involves the student’s ability to listen to directions and explanations, participate in class discussions, focusing on different parts of the activity and stimuli at various moments in the lesson, and the ability to sustain an undisclosed amount of time dedicated to working and thinking on an activity for any given time range. Paying attention is learned.

Conceptual
Conceptual learning is when students use the previous learning experiences, ideas and concepts to understand and develop the new meaning of the new content by recognizing the connection and relationship of the old and new “building block concepts.”

Memory
Both long-term and short-term memory play essential roles in the learning of mathematics. Math is a conceptual subject consisting of connections and relationships.

Reasoning
Reasoning is the capacity to reflect on work, evaluate it, and then adapt, as needed.

Executive Functioning
Executive functioning is closely aligned to behavior and is judged by the strength of:
- self-awareness (self-directed attention),
- inhibition (self-restraint),
- non-verbal working memory (the ability to hold things in your mind),
- verbal working memory (self-speech),
- emotional self-regulation (learning to use words, images and your own self-awareness to process and alter how you feel about things),
- self-motivation (how well you motivate yourself to complete a task) and
- planning and problem solving (how you organize information in your brain).

Instructional Climate and Student Mindsets
Positive instructional climate and student mindsets can be developed through experiences obtained from physical location, ideas, and expectations of those within the environment. Classroom opportunities that demonstrate an awareness of and embrace all students’ ability to achieve success in mathematics can increase student autonomy and academic outcomes. Student autonomy encourages students to speak from their mathematical perspective and advocate for their own reasoning. Positive school climate leads to encouraging and supportive classroom environments that ultimately lead to growth mindsets and increased agency for each and every student.
**Language Processing**
Language processing involves students’ ability to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing. Language processing refers to the way humans use words to communicate ideas and understandings, and how such communications are processed and understood.

**Language Proficiency**
Mathematics language proficiency is measured in terms of interpretive and expressive language skills at the word/phrase/representation, sentence and discourse dimensions.

**Mathematics Calculation**
Mathematical calculations are basic computational skills (with and without the appropriate grade level calculator) and basic graphing skills that support the mathematical language, when solving contextual, mathematics problems. As a result of focusing on conceptual understanding, over time students develop procedural fluency (accurate, efficient, and flexible) and learn to make sense of and solve problems using automaticity of facts rather than rote recall.

**Other Exceptionalities**
Students with other exceptionalities may experience physical, social or emotional barriers that have the potential to influence how they engage in the learning experiences. Instructional opportunities should be limitless for all students to assist them with meeting high academic standards and developing mathematical reasoning and conceptual understanding of the content expectations.

**Problem-Solving**
Problem-solving is a cognitive process that involves students learning mathematics through contextual, mathematical problems and models. The contexts and models allow students to build meaning for the concepts so they can fully understand abstract concepts. Students are expected to use problem-solving strategies and develop these skills over time as they progress in the K - 12 curriculum.

**Visual-Spatial Processing**
Visual-spatial processing is a student’s ability to perceive, analyze, and understand visual information and communicate with the teacher and peers.
**English Language Proficiency for English (as a 2nd language) Learners**

Students use their **emerging, developing, or expanding** 2nd language (English) to make meaning and engage with mathematics understandings. The student observations presented here will concentrate on students at the emerging, developing, and expanding levels of English language proficiency (ELP) as measured by the ACCESS for ELLs (CPL levels 2, 3, and 4) which describe most EL students. Recently arrived ELs at ELP 1 comprise <10% of the EL student population, and those who have achieved ELP 5 have exited the EL status and ESOL program services. It is expected that teacher’s will identify their student’s level of English proficiency to select the appropriate teacher actions.

<table>
<thead>
<tr>
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</table>
| **Observations**     | The teacher will support comprehension and make mathematics content linguistically comprehensible by providing temporary scaffolds that include:  
• storytelling in student’s home and community context or language to pose word problems,  
• making language visible through teacher acting story out or modeling commands for students’ total physical response,  
• visual representation of word problems using manipulatives or drawings to support sense-making, and  
• multiple opportunities for structured peer interactions or conversations (pairs or triads) to negotiate meaning using listening cues and highlighted or reduced text.  | The student will make sense of word problems in English through acting out math stories, using mathematics manipulatives, creating visual representation or drawings, and physically responding to listening cues in structured peer interactions and from repeated teacher modeling.  | Bilingual or dual language mathematics storybooks in primary languages represented by students’ families.  
Mathematics storybooks in English with reduced text and increased pictures, graphics, and visuals.  
Mathematics manipulatives and other mathematics realia for grade level.  
Vocabulary games  
Number Talks  |
| **Observations**     | The teacher will support communication by providing temporary scaffolds that include:  
• Making language visible through  | The student will record mathematical reasoning through simple sentence journaling, picture writing, or structured peer interactions  | Mathematical Language Routines to Optimize Student Output  
Scaffolds:  
• Realia  |
| revelations of silence followed by **emerging** English language to communicate mathematical ideas using unclear pronunciation,  |  |  |  |
| Gestures, or words/phrases in primary language (Expressive Language Skills: Speaking and writing skills) | Repeated teacher modeling of mathematics language and think-aloud, use of mathematics manipulatives or *realia* to support communication, and multiple opportunities for structured peer interactions or conversations (pairs or triads) to practice English using pictures, manipulatives, realia, a word bank and/or sentence frames. | To describe mathematical thinking to others. | • Pictures  
• Math manipulatives  
• Grade-level wordbanks  
• Grade-level sentence frames  
• Partner Listen/Talk  
• Digital resources |
|---|---|---|---|
| Observations reveal developing comprehension of mathematical text in English, such as word problems (Interpretive Language Skills: listening, viewing, and reading) | The teacher will support comprehension and make mathematics content linguistically comprehensible by providing temporary scaffolds that include: making language visible through repeated teacher modeling and think-aloud, visual representation of word problems using manipulatives or drawings, and multiple opportunities for structured peer interactions or conversations (pairs or triads) to negotiate meaning using listening cues and highlighted or reduced text. | The student will make sense of word problems through using mathematics manipulatives, creating visual representation or drawings, and physically responding to listening cues in structured peer interactions and teacher modeling. | Vocabulary games  
Math manipulatives for grade level.  
| Observations reveal developing English language to communicate mathematical ideas (Expressive Language Skills: Speaking and writing skills) | The teacher will support communication by providing temporary scaffolds that include: making language visible through repeated teacher modeling of mathematical language and think-aloud, use of charts, or graphic organizers to support sense-making, and multiple opportunities for structured peer interactions or conversations (pairs or triads) to negotiate meaning using listening cues and highlighted or reduced text. | The student will record mathematical reasoning in English through journaling or structured peer interactions to inform, narrate, argue, or explain mathematical thinking to others. | Mathematical Language Routines to Optimize Student Output  
• *Stronger & Clearer Each Time*  
• *Convince Yourself, a Friend, a Skeptic*  
• *Info Gap Cards*  
Number Talks  
Scaffolds:  
• Graphic organizers  
• Word banks |
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<tr>
<th>Observations reveal <strong>expanding</strong> comprehension and interpretation of mathematical explanations in English, such as problem-solving steps or evaluating a pattern or structure that follows a given rule. <em>(Interpretive Language Skills: listening, viewing, and reading)</em></th>
<th>The teacher will support comprehension and make mathematics content linguistically comprehensible by providing temporary scaffolds that include:  - making language visible through repeated teacher modeling and think-aloud,  - visual representation of word problems using charts, graphs, visual data, and  - multiple opportunities for structured peer interactions or conversations (pairs or triads) to negotiate meaning using listening cues and highlighted or reduced text.</th>
<th>The student will make sense of word problems in English through using manipulatives, and creating visual representation, data charts, data graphs, and responding to listening cues in structured peer interactions and teacher modeling.  The student will expand their comprehension of short text, synonyms, and antonyms, expanded noun groups with prepositional phrases, and multiple related simple sentences regarding mathematical reasoning and calculations.</th>
<th>Language Demand in Mathematics Lessons (LDML) tool at <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a>  Framework for Analyzing Word Problems: Guiding Questions, Language Demands, and Tasks for Teachers – <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a> <em>(Access code: ELL14118)</em></th>
</tr>
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<tr>
<td>Observations reveal <strong>expanding</strong> language to construct mathematical explanations in English. <em>(Expressive Language Skills: Speaking and writing skills)</em></td>
<td>The teacher will support students' communication of mathematical explanations by providing temporary scaffolds that include:  - making English language visible through repeated teacher modeling of language connectors to order steps in a mathematical process (first, next, then), language to indicate causal relationship (because, so, that, means, as a result), abstract, generalized, or multi-meaning noun groups to add precision to mathematical descriptions (operation, associative property, area formula, function), past tense verbs and thinking verbs to recount</td>
<td>The student will construct mathematical explanations in English that introduce a concept, share a solution with others, describe data and steps to solve a problem, and state the reasoning they used to generate the solution.  The student will participate in structured peer interactions that convey intended purposes to others for mathematical thinking.</td>
<td>Mathematical Language Routines to Optimize Student Output  - Collect &amp; Display  - Gather &amp; Show Student Discourse  - Critique, Correct, and Clarify  Scaffolds:  - Mathematics word banks  - Sentence frames  - Partner Talk  - Digital resources  Language Demand in Mathematics Lessons (LDML) tool at <a href="http://www.nctm.org/more4u">www.nctm.org/more4u</a> <em>(Access code: ELL14118)</em></td>
</tr>
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</table>
- steps and reasoning aloud,
- use of visuals (charts, graphs, diagrams, manipulatives, drawings) to support approach and/or solution,
- multiple opportunities for structured peer interactions or conversations (pairs or triads) to negotiate meaning using manipulatives, data graphs/charts, sentence frames, a word bank of mathematical terms.
Additional Resources

Establish mathematics goals to focus learning.
- [10 Common Challenges Article](#) This article describes various ways teachers can make decisions based on their instructional goals.
- [Resources to Increase Students Interest](#) This is a website with different resources to increase student interest.
- [Standards Based Math Rubric](#) This resource is the learning progression for Math Standards.

Implement tasks that promote reasoning and problem solving.
- [Understanding Word Problems in Mathematics](#) This resource provides strategies to help students understand and solve word problems.
- [Activities That Boost Spatial Reasoning](#) This website provides strategies to boost visual spatial reasoning.
- [Graph Paper Math Intervention](#) This website describes how graph paper can be used as a math intervention.
- [Step by Step Graphic Organizer](#) This resource is a Graphic Organizer to help students complete multi-step tasks/word problems.

Use and connect mathematical representations.
- [Information on KWL and KNWS Charts](#) This resource explains the use of KWL and KNWS charts in Mathematics.

Facilitate meaningful mathematical discourse.
- [Mathematical Language Brainstorm](#) This resource is a student self-assessment/brainstorm graphic organizer.
- [Math Journal Examples](#) This resource provides many journal prompts and questions that elicit conceptual thinking and creativity.
- [Effective Word-Problem Instruction: Using Schemas to Facilitate Mathematical Reasoning](#)

Pose purposeful questions.
- [GSE Effective Instructional Practices Guide](#) This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices.
- [GA Frameworks Video:MGSE2 NBT 9](#) Video showing series of questions the teacher asks that connect previous learning to new knowledge.
- [Is Your Answer Reasonable?](#)
This lesson gives the teacher suggestions for guiding students in determining reasonableness.

### Build procedural fluency from conceptual understanding.
- **Early Number and Counting Video**
  This is part of the Progression Series from Graham Fletcher that explains the progression of early number and counting skills.

- **Base Ten Block Place Value Activities**
  This resource is a Video explaining base-ten value activities.

- **Practice Using Base Ten Blocks**
  This resource is a handout of base-ten block manipulatives.

- **Virtual Base Ten Blocks Manipulatives**
  This resource is a link of digital mathematical manipulatives.

- **Concepts for Base Ten Understanding**
  This resource is an article discussing base-ten understanding.

### Support the efforts of learning in learning mathematics.
- **Understanding Word Problems in Mathematics**
  This resource provides strategies to help students understand and solve word problems.

- **Three Reads Using a Problem Stem**
  Provides definition of Three Reads, background information, and sample Three Reads lessons

### Elicit and use evidence of student thinking.
- **Student Reflection**
  This resource is a student self-assessment/ reflection graphic organizer.

- **Student Rubric**
  This resource is a self-assessment rubric.
References


Celedón-Pattichis, S., & Ramírez, N.G. (2013) Advancing mathematics education for ELs – Beyond good teaching (NCTM)


