Georgia Mathematics Strategies Toolkit to Address Learner Variability for Grades 6 - 8

GaDOE
Georgia Department of Education

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Introduction

“If the goal in mathematics teaching and learning is to support student success with mathematical proficiency, then we must be explicit about using instructional routines that focus on student engagement in activities that support reasoning and sense making, communication with and about mathematical ideas, making meaningful connections, building procedural fluency from conceptual understanding…”

- Thinking about Instructional Routines in Mathematics Teaching and Learning

Within this toolkit, educators will find observations of student behavior for each of the 14 identified areas for addressing learner variability. Aligned to each observation of student behavior, are evidence-based, research-based strategies intended to strengthen students’ ability in mathematics. Support resources are provided to assist educators with implementing the strategies.

- Behavior
- Cognitive Processing
  - Attention
  - Conceptual
  - Memory
  - Reasoning
- Executive Functioning
- Instructional Climate and Student Mindsets
- Language Processing
- Language Proficiency
- Mathematics Calculation
- Other Exceptionalities
- Problem-Solving
- Visual-Spatial Processing

Looking for more evidence-based, researched based practices for mathematics? Please visit gadoe.org/mathematics.
Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning. (EMTP 1)
2. Implement tasks that promote reasoning and problem solving. (EMTP 2)
3. Use and connect mathematical representations. (EMTP 3)
4. Facilitate meaningful mathematical discourse. (EMTP 4)
5. Pose purposeful questions. (EMTP 5)
6. Build procedural fluency from conceptual understanding. (EMTP 6)
7. Support efforts of learning in mathematics. (EMTP 7)
8. Elicit and use evidence of student thinking. (EMTP 8)

Mathematical Practices

1. Make sense of problems and persevere in solving them. (MP.1)
2. Reason abstractly and quantitatively. (MP.2)
3. Construct viable arguments and critique the reasoning of others. (MP.3)
4. Model with mathematics. (MP.4)
5. Use appropriate tools strategically. (MP.5)
6. Attend to precision. (MP.6)
7. Look for and make use of structure. (MP.7)
8. Look for and express regularity in repeated reasoning. (MP.8)
**Behavior**

**Mathematics Connection:** When students develop productive mathematical behavior, they are confident in their ability to attain proficiency and outstanding performance on an assignment. They are not afraid to ask questions or seek assistance when needed, and they are confident and display limited frustration when grappling with challenging problems.

**Student Learning Expectations:** Students should have opportunities to develop a growth mindset and transform to show a mathematical mindset by gaining confidence, taking instructional risks, and asking questions for clarity on challenging activities to achieve at their optimal performance.

### Behavior Observations

<table>
<thead>
<tr>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</table>
| Observations reveal extended time off task and continuously talks during routine instruction. | The teacher will motivate students’ learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. (EMTP 2 & 7) | The student will make sense of problems and persevere in solving them. (MP.1) - The student will actively participate in learning experiences and task completion will improve. - The student will remain on task during routine instruction. | **PBIS Strategies**
Examples of positive behavior strategies.  
**Guide for Effective Mathematics Instruction**
These mathematics toolkits provide strategies to identify gaps in learning and evidence-based strategies to address the gaps. |

| Observations reveal signs of feeling overwhelmed resulting in shuts down during instruction. | The teacher will support the efforts of learning in mathematics. (EMTP 7) - The teacher will provide students with manageable tasks. - The teacher will anticipate where students may need additional supports during a lesson and be prepared to support them in the efforts of learning. | The student will reason abstractly and quantitatively while working in chunks to complete the task. (MP.1 & 2) - The student will utilize mathematics comprehension strategies to solve problems. | **3 Reads Instructional Routine**
Designed to develop students’ ability to make sense of problems by deconstructing the process of reading mathematical situations. |

| Observations reveal distracting behavior during group work. | The teacher will facilitate discourse among students by positioning them as authors of ideas, who explain and defend their approaches. (EMTP 4) - The teacher will assign the student a group role. | The student will work collaboratively with peers and participate in mathematical discussions. (MP.1, 2 and 3) | **Group Work**
An article with tips for implementing group work roles.  
**HLP 17: Use Flexible Grouping**
High-leverage practices for well-designed strategic and adaptable instruction. |
Cognitive Processing: Attention, Conceptual, Memory, Reasoning

Mathematics Connection: Cognitive processes include developmental processes such as attention, memory, reasoning, and conceptual processes; when used by students every day as a support strategy, these strategies will allow students to develop profound connections of the concepts, be able to recognize relationships, and perform cognitive procedures with situational-based problems. Once students make the connection, they can transfer that new acquired thinking to problem solving.

Student Learning Expectations: Students should have opportunities to use cognitive process strategies as a learning support tool to bridge the prior ideas with the new learning concepts. Students are now fully equipped with the ability to perform the procedures to arrive at a solution.

Cognitive Processing: Attention

Mathematics Connection: Increasing complexity of mathematics content and tasks require extended attention spans. Maintaining attention can be challenging if a student is uninterested or the task seems too challenging.

Student Learning Expectations: Students should have opportunities to focus on a portion of any given information to concentrate on the relevant content, while using cognitive attention processes to remain focused on the pertinent information.

<table>
<thead>
<tr>
<th>Attention</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
<tbody>
<tr>
<td>Observations</td>
<td>The teacher will support the efforts of learning by providing multiple scaffolds through the learning experiences. (EMTP 7)</td>
<td>The student will monitor and evaluate their progress and change course if necessary while solving problem. (MP.1) • The student will submit various components of the project at designated points.</td>
<td>Using Rubrics in Middle Grades Checklists are provided for teachers to assess project items in increments. HLP 13: Adapt Curriculum Tasks and Materials for Specific Learning Goals High-leverage practices for well-designed strategic and adaptable instruction.</td>
</tr>
<tr>
<td>Observations reveal a struggle to maintain the attention to complete a long assignment/project.</td>
<td>• The teacher will chunk the project into small check points. • The teacher will provide a grading rubric.</td>
<td></td>
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<tr>
<td>Observations reveal a lack of interest in assignments.</td>
<td>The teacher will implement tasks that promote reasoning and problem solving. (EMTP 2) • The teacher will implement reach-informed, evidence-based strategies to increase engagement. • The teacher will provide the student with choices.</td>
<td>The student will make sense of problems, apply reasoning strategies and persevere in solving problems. (MP.1 &amp; 2) • The student will choose assignment(s) of their choice that displays their understanding of the standard.</td>
<td>Guide for Effective Mathematics Instruction These mathematics toolkits provide strategies to identify gaps in learning and evidence-based strategies to address the gaps. HLP 18: Use Strategies to Promote Active Student Engagement High-leverage practices for well-designed strategic and adaptable instruction.</td>
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</table>
Cognitive Processing: Conceptual

Mathematics Connection: Conceptual understanding involves meaningfully learned and well-integrated knowledge about mathematics. Students should gain many logical connections between specific concepts and ideas.

Student Learning Expectations: Students should have opportunities to use conceptual processing to help make sense of contextual, mathematical problems to gain a deeper understanding of the integrated knowledge and the connections of the big idea concepts.

<table>
<thead>
<tr>
<th>Conceptual</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</table>
| Observations reveal a struggle to solve abstract problems. | The teacher will provide opportunities to use and connect representations to make sense of quantities and their relationships. (EMTP 3)  
- The teacher will make connections to familiar context by purposefully activating background knowledge. | The student will routinely seek patterns and structures to model and solve contextual, mathematical problems. (MP 7)  
The student will use repeated reasoning to understand algorithms and make generalizations about patterns. (MP 8) | Connecting Representations  
NCTM presentation on connecting representations.  
Contemplate then Calculate  
An instructional routine that develops students’ capacity to attend to mathematical structure when problem solving. |
| Observations reveal a struggle to make generalizations. | The teacher will provide opportunities for students to demonstrate their flexibility in representing mathematics in several ways. (EMTP 3)  
- The teacher will explore patterns in different ways using appropriate tools.  
The teacher will facilitate mathematical discussions about math problems. (EMTP 4) | The student will make connections between and among concepts by utilizing manipulatives when solving contextual, mathematical problems. (MP 4)  
The student will recognize the significance in concepts and models and use the patterns or structure for solving related problems. (MP 7) | Recognizing Repetition  
An instructional routine that supports the difficult road to generalizing problem situations.  
SMP Observation Tool  
Mathematical Practices 7 and 8. |
| Observations reveal a lack of understanding of mathematical relationships and connections. | The teacher will allot substantial instructional time for students to use, discuss, and make connections among representations. (EMTP 3)  
The teacher will explicitly make connections to previous learning experiences during each day’s lesson or task. (EMTP 1) | The student will explain connections between their mathematical understanding and reasoning with drawings, diagrams, and other representations. (MP 2, 3, 7 & 8) | Connecting Representations  
An instructional routine that positions students to think structurally as they connect two representations by articulating the underlying mathematics. |
| Observations reveal a lack of knowledge of the | The teacher will provide students with different representations of the | The student will make sense of mathematical vocabulary words used in contextual | 3 Reads  
Instructional routine supporting problem solving. |

SMP Observation Tool
Cognitive Processing: Memory

Mathematics Connection: Using rote memory to recall facts or remember the steps of an algorithm does not yield long-term learning. Students should have opportunities to use conceptual learning strategies that will lead to committing the basic computational facts to memory.

Student Learning Expectations: Students should have opportunities to use knowledge from their memories to perform calculations and procedures, identify geometric figures, and demonstrate basic graphing skills by using visual-spatial and numerical representations to make sense of contextual, mathematical problems to help with sustaining long-term memory.

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<th>Memory</th>
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<td><strong>Observations</strong></td>
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</table>
| Observations reveal a struggle to remember previously taught mathematics skills and concepts. | The teacher will activate prior knowledge by questioning before, after, and during the lesson. (EMTP 5)  
- The teacher will ask questions that build on, not do not take over or funnel student thinking.  
- The teacher will design ways to elicit and assess students’ abilities to use representations meaningfully to solve problems. (EMTP 3 & 8) | The student will use mathematical structures to make connections between and among mathematical ideas. (MP.7 & 8) | Prior Knowledge Warm-Up Activities  
In mathematics lessons, most warm-up activities that activate prior knowledge are generated either by mathematical task or a discussion prompt you provide for the students as class begins. |
| Observations reveal a struggle remembering mathematics terms/vocabulary or cannot apply understanding of the terms when solving problems. | The teacher will focus students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation or concept. (EMTP 3)  
- The teacher will introduce term through contextual situations and provide a resource guide with mathematics terms.  
- The teacher will provide graphic organizer math terms such as Frayer Squares. | The student use reasoning about mathematics terms in context to apply within problem solving situations. (MP.2)  
- The student will create a resource list of math terms utilizing the Frayer Squares.  
- The student will use precise mathematical language when explaining thinking. (MP.6) | Frayer Model  
An example of the Frayer model is provided with commentary to assist teachers with implementing this strategy in the classroom.  
Five Effective Strategies for Teaching Vocabulary  
This website provides techniques for teaching vocabulary which will assist math teachers with math terminology. |
Observations reveal a struggle to solve multi-step mathematics problems or remember the sequence of steps in a problem-solving situation.

| The teacher will use and connect mathematical representations to help students make sense of contextual, mathematical problems. (EMTP 3)  
  • The teacher will model skill/concept using examples and non-examples.  
  The teacher will chunk information into smaller steps. (EMTP 7)  
  The teacher will provide clear, multi-sensory modeling of a mathematics skill/concept utilizing cues. | The student will utilize an efficient strategy and apply that strategy to new situations. (MP.7)  
  • The student will remember concepts and skills due to the multi-sensory instruction (e.g., differentiation, mnemonics). | Connecting Representations NCTM presentation on connecting representations.  
  Chunking This website explains chunking.  
  Multi-Sensory Instruction for Mathematics 10 ways to utilize multi-sensory instruction in the mathematics classroom. |

### Cognitive Processing: Reasoning

**Mathematics Connection:** Mathematical reasoning enables students to make use of the concepts and skills in the lesson by gaining access to understanding. Students evaluate the situations, select problem-solving strategies, draw conclusions, develop and describe solutions and situational context. When students apply mathematical reasoning, they can reflect on the contextual, mathematical problems.

**Student Learning Expectations:** Students should have opportunities to critique and explain their thinking to make sense of the problem and the reasoning of others as they engage in contextual, mathematical situations.

<table>
<thead>
<tr>
<th>Reasoning</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</table>
| **Observations** reveal difficulty connecting new information to background knowledge. | The teacher will provide opportunities to solve problems that build upon and extend prior knowledge. (EMTP 2)  
  • The teacher will use think aloud strategies to make connections between mathematics concepts. | The student will make sense of problems and routinely seek patterns or structures to model and solve problems. (MP.1 & 7) | Thinking Aloud in Mathematics  
  This website explains the benefits of modeling “Think Alouds”. |
| **Observations** reveal difficulty explaining their thinking. | The teacher will implement tasks that promote reasoning and problem solving. (EMTP 2)  
  • The teacher will utilize language routines to support students in formulating their explanations. | The students will refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. (MP.3)  
  • The student will use sentence frames to help | Mathematical Language Routines  
  A presentation from Illustrative Mathematics about various language routines. |
| Observations reveal a struggle to solve multi-step mathematics problems or remember the sequence of steps in a problem-solving situation. | The teacher will use and connect mathematical representations to help students make sense of contextual, mathematical problems. (EMTP 3)  
- The teacher will model skill/concept using examples and non-examples.  
The teacher will chunk information into smaller steps. (EMTP 7)  
The teacher will provide clear, multi-sensory modeling of a mathematics skill/concept utilizing cues. | The student will utilize an efficient strategy and apply that strategy to new situations. (MP.7)  
- The student will remember concepts and skills due to the multi-sensory instruction (e.g. differentiation, mnemonics). | Connecting Representations  
NCTM presentation on connecting representations.  
Chunking  
This website explains chunking.  
Multi-Sensory Instruction for Mathematics  
10 ways to utilize multi-sensory instruction in the mathematics classroom. |
Executive Functioning

**Mathematics Connection:** Executive functioning is a set of cognitive mental competencies that students use to organize information and put it into use while working on mathematical problems, visualize problems from multiple angles, come up with different ways to solve them, and control attention and behavior to manage emotions when encountering the efforts of learning.

**Student Learning Expectations:** Students should have opportunities to make sense of problems and persevere in solving them while making use of structure and reasoning about the repeated use of structure. Students will also be expected to model with mathematics.

<table>
<thead>
<tr>
<th>Executive Functioning</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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<tbody>
<tr>
<td><strong>Observations</strong></td>
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</table>
| completion of assignments with haste without taking time to absorb expectations. | The teacher will establish mathematics goals. (EMTP 1)  
- The teacher will identify learning targets at the beginning of each lesson.  
- The teacher will break down tasks into smaller components.  
- The teacher will provide a checklist for each task component. | The student will make sense of a given problem. (MP.1)  
- The student will assess what they know and what they anticipate needing help with when presented with a new task.  
- The student will attend to precision. (MP.6)  
- The student will calculate accurately and efficiently. | Learning Intentions with Success Criteria  
This document provides guidance on how to develop and effectively implement learning targets.  
Strategies to Support Executive Functioning Challenges: Sequential Checklists  
This website provides examples of sequential math checklists. |
| Observations reveal a struggle to self-monitor learning. | The teacher will provide time for students to grapple with tasks and ask questions that scaffold students’ thinking without doing all of the thinking and problem solving for them. (EMTP 7)  
The teacher will provide opportunities for reflection throughout the lesson. (EMTP 8) | The student will monitor and evaluate progress and change course as they analyze answers. (MP. 1 & 6) | Treating Reflection as a Habit, Not an Event  
Simple strategies for making reflection a regular routine.  
9 Steps for Breaking Down Assignments  
This website describes effective ways to break a lesson into manageable parts. |
# Instructional Climate and Student Mindsets

**Mathematics Connection:** A student's mindset and disposition can influence how they understand, connect, and recognize relationships in mathematics. A connection between mathematical mindsets and learning must be addressed if mathematics is going to be accessible to all students. Students' mathematical understanding is developed over time; therefore, it is important to recognize how students' understanding of their abilities and the instructional climate can affect mathematics conceptual learning.

**Student Learning Expectations:** Students should have opportunities to see themselves as numerate contributors to the world around them and beyond and think deeply about mathematics as they prepare to enter the future workforce with the critical thinking and reasoning skills necessary for success in both the local and global workforces.

<table>
<thead>
<tr>
<th>Instructional Climate and Student Mindsets</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations reveal student has a negative mathematics disposition. Ex: “I can’t do math.”, “I’m not a math person.”</td>
<td>The teacher will motivate student learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. (EMTP 2) The teacher will identify what counts as evidence of student progress toward mathematics learning goals. (EMTP 8) The teacher will provide various examples of how mathematics can be applied to a student’s daily life to form connections between student’s interests and mathematical learning. (EMTP 2)</td>
<td>The students will form connections with daily aspects of their lives (and classmates’ daily lives) and mathematical learning through teacher’s mathematical topic examples. (MP.1) The student will think deeply about the mathematics using critical thinking and reasoning skills necessary for success. (MP.1, 2 &amp; 3) The student will check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?” (EMTP 1)</td>
<td>5 Teaching Strategies to Build Math Confidence This website discusses 5 growth mindset strategies for students who lack math confidence.</td>
</tr>
<tr>
<td>Observations reveal a sense of irrelevance of mathematical concepts discussed in class.</td>
<td>The teacher will incorporate real-world connections into math practices and elicit responses from students in ways that incorporate a variety of ways of thinking, learning, and communicating. (EMTP 2, 4 &amp; 8) The teacher will develop students' understanding of mathematics in by using authentic, real-life tasks that are centered around real phenomena. (EMTP 4)</td>
<td>The student will solve mathematical problems centered around real-life experiences to develop their mathematical understanding. (MP.1, 2, 6 &amp; 7)</td>
<td>7 Real-World Math Strategies How do you use the real world to teach math?</td>
</tr>
</tbody>
</table>
## Language Processing

**Mathematics Connection:** Language processing is essential to mathematical discourse. Mathematical discourse is about the exchange of mathematical ideas, including ways of representing, thinking, talking, agreeing, and disagreeing. Language processing involves students’ ability to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing.

**Student Learning Expectations:** Students should have opportunities to express mathematical ideas with precision and clarity.

<table>
<thead>
<tr>
<th>Language Processing</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
</table>
| Observations reveal inefficient use of words to communicate ideas and understandings. | The teacher will use language routines to support students’ communication of ideas and understandings. (EMTP 4) | The student will utilize strategies to organize their ideas and understanding allowing for the communication of said ideas and understandings to be comprehended clearly by others. (MP.3) | **Mathematical Communication**  
This “Mathematical Communication” is a collection of resources for engaging students in writing and speaking about mathematics.  
**Mathematical Language Routines**  
Presentation from Illustrative Mathematics about various language routines. |
| Observations reveal a struggle to ask clarifying questions. | The teacher will model how to construct an effective question in math. (EMTP 4 & 5)  
The teacher will make real-time decision on how to respond to students with questions and prompts that probe, scaffold, and extend. (EMTP 8) | The student will ask questions, respond to and give suggestions to support the learning of themselves and their classmates. (MP.1, 2, 3 & 6)  
The students will start to formulate effective questions for the teacher and peers. (MP.3) | **8 Ways to Pose Better Questions in Math Class**  
This article includes “8 Ways to Pose Better Questions in Math Class” for teachers and students.  
**PowerUp What Works: Thinking Aloud**  
Best practices for using technology to support reasoning by focusing on student thinking. |
**Language Proficiency**

**Mathematics Connection:** Language proficiency is essential to mathematical literacy and comprehension. Language proficiency involves students’ ability to understand and use the language of mathematics to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing.

**Student Learning Expectations:** Students should have opportunities to precisely use mathematical language to explain their thinking and reasoning in mathematics.

<table>
<thead>
<tr>
<th>Language Proficiency</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
<tbody>
<tr>
<td>Observations reveal an inability to use the language of mathematics in writing to communicate mathematical ideas with precision and clarity.</td>
<td>The teacher will model the use of the language of mathematics in their writing as well as how to claim an answer and support it. (EMTP 4)</td>
<td>The student will justify their answers with the use of the language of mathematics in their writing. (MP.2 &amp; 3)</td>
<td>Promoting Mathematical Argumentation This website provides examples and recommendations for teachers of how to promote mathematical argumentation. Math Language Routines Illustrative Mathematics blog post on using math language routines with English Language Learners.</td>
</tr>
<tr>
<td>Observations reveal an inability to use the language of mathematics orally to articulate mathematical ideas with precision and clarity.</td>
<td>The teacher will model the use of the language of mathematics in their oral response as well as how to claim an answer and support it. (EMTP 4 &amp; 5) • The teacher will model usage of sentence starter-frames to use the mathematical language.</td>
<td>The student will justify their answers with the use of the language of mathematics in their oral response. (MP.3)</td>
<td>Math Language Routines Illustrative Mathematics blog post on using math language routines with English Language Learners. Sentence Stems to Encourage Math Talks This website provides “25 Sentence Stems to Encourage Math Talk” along with a guide to how to use the sentence stems in a math class.</td>
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# Mathematics Calculation

**Mathematics Connection:** Mathematical calculation involves using mathematical or logical reasoning to solve a problem. When demonstrating math computation, students understand and use mathematical language, learn processes and procedures for solving math problems, and reflect on solutions to problems to determine if they make sense.

**Student Learning Expectations:** Students should have opportunities to show growth development in number sense to perform math calculations fluently by using the connections of understanding between visual-spatial and numerical representations.

<table>
<thead>
<tr>
<th>Mathematics Calculations</th>
<th>Observations</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
<th>Support Resources</th>
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</thead>
<tbody>
<tr>
<td>Observations reveal difficulties maintaining precision when solving mathematical problems.</td>
<td>The teacher will connect student-generated strategies and methods to more efficient procedures as appropriate. (EMTP 6 &amp; 7)</td>
<td>The student will increase their number sense and math calculations as they engage in number talk activities. (MP.1)</td>
<td>Secondary Number Talks This website provides various examples of number talk activities and how to implement these activities in the classroom.</td>
<td></td>
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<tr>
<td>Observations reveal an overreliance on rote memorization when solving contextual, mathematical problems.</td>
<td>The teacher will ask probing questions to elicit conceptual understanding of the mathematical procedures. (EMTP 5 &amp; 8) The teacher will provide opportunities for students to solve non-routine problems. (EMTP 2)</td>
<td>The student will use repeated reasoning to make generalizations and connections when solving problems. (MP.7 &amp; 8) • The student will model and solve contextual problems and construct other examples and models that confirm a generalization or connection of mathematical ideas.</td>
<td>GSE Effective Instructional Practices Guide This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices.</td>
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<tr>
<td>Observations reveal difficulty with computation due to misreading mathematical symbols.</td>
<td>The teacher will build procedural fluency from conceptual understanding by asking students to discuss and explain why the procedures used work to solve problems. (EMTP 6) The teacher will use tools to discuss and make connections among representations. (EMTP 4)</td>
<td>The student will use pictures and numbers to represent solutions to mathematical, contextual problems. (MP.4) The student will justify and explain thinking when solving problems. (MP.1 &amp; 2)</td>
<td>Contemplate then Calculate An instructional routine designed to shift attention away from mindless calculations and toward necessary structural interpretations of mathematics. This routine fosters structural thinking, math practice 7.</td>
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Other Exceptionalities

**Mathematics Connection:** Instruction must incorporate strategies and supports providing access to the content within the general education curriculum for mathematics, rich learning tasks that allow for the implementation of the Mathematical Practices and the building of proficiency in mathematics for all learners through sustained and deep engagement in practices that include problem solving, reasoning, and critiquing.

**Student Learning Expectations:** Students should have opportunities to meet high academic standards and demonstrate the level of mathematical reasoning needed to fully develop their conceptual understanding and procedural fluency.

<table>
<thead>
<tr>
<th>Other Exceptionalities</th>
<th>Teacher Actions</th>
<th>Student Actions</th>
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</table>
| Observations reveal a struggle with endurance during classroom assignments. | The teacher will reduce the length of the assignment. (EMTP 7)  
- The teacher will provide the student with frequent breaks. | The student will solve problems and persevere in solving them. (MP.1)  
- The student will complete the assignment in parts.  
- The student will self-advocate for breaks when he or she feels fatigue. | Homework versus Shortened Assignment  
This article explains the difference between homework and reducing the length of an assignment. |
| Observations reveal a struggle to verbally communicate with teacher and peers in the classroom. | The teacher will provide the student with augmentative communication device. (EMTP 5) | The student will use the device which allows the student to communicate. | AAC  
This website explains augmentative communication devices and provides resources for teachers and parents. |
| Observations reveal trouble accessing the curriculum during routine instruction due to proximity. | The teacher will provide environmental adaptations such as preferential seating. | The student will sit close to the teacher to enable him/her to enhance their learning. | Inclusive Teaching  
This website provides teaching and assessment strategies for students who are deaf/hard of hearing. |
| Observations reveal a struggle to see the whiteboard or instructional material. | The teacher will provide preferential seating to the student.  
The teacher will provide enhanced print on classroom assignments. | The student will access the information on the board or instructional materials to enable him/her to enhance their learning. | Adaptations for student with Visual Impairments  
This website provides information on mobility adaptations for all learners. |
**Problem-Solving**

**Mathematics Connection:** Problem-solving is a cognitive process that involves students learning mathematics through contextual, mathematical problems and models. The contexts and models allow students to build meaning for the concepts so they can fully understand abstract concepts.

**Student Learning Expectations:** Students should have opportunities to use mathematical reasoning to make sense of the content, extract information and explore new concepts as they persevere in solving contextual, mathematical problems.

| Problem Solving |
|-----------------|-----------------|-----------------|-----------------|
| **Observations** | **Teacher Actions** | **Student Actions** | **Support Resources** |
| Observations reveal difficulty connecting the conceptual and abstract concepts of mathematics. | The teacher will focus students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. (EMTP 4) The teacher will use visual models to support students’ understanding of general methods. (EMTP 6) | The student will look for and make use of structure when transfer knowledge from concrete to representation to abstract phases of learning. (MP.7) The student will explain and justify their thinking using words, objects, and drawings. (MP.2) | Error Analysis for Mathematics This website provides access to problems that require students to justify their thinking. Concrete Representational Abstract A three-step instructional approach that has been found to be highly effective in teaching math concepts. |
| Observations reveal difficulty understanding mathematical terminology when solving problems. | The teacher will facilitate meaningful mathematical discourse. (EMTP 6) The teacher will purposefully select and explicitly teach key vocabulary in context, using visuals, modeling, and gestures. (EMTP 4) | The student will attend to precision while using mathematical terminology. (MP.6) ● The student will participate in active and significant use of key vocabulary in a variety of learning activities. | Math and Special Education Blog Use these 5 actionable strategies today to grow your students’ math vocabulary. |
| Observations reveal a struggle to solve multi-step contextual, mathematical problems. | The teacher will anticipate what students might struggle with during a lesson and be prepared to support them productively through their efforts. (EMTP 7) ● The teacher will chunk information into smaller steps. | The student will utilize an efficient strategy and apply that strategy to new situations. (MP.7) ● The student will remember concepts and skills due to the multi-sensory instruction (e.g. differentiation, mnemonics). | Guide for Effective Mathematics Instruction These mathematics toolkits provide strategies to identify gaps in learning and evidence-based strategies to address the gaps. Chunking This website explains chunking. |
Visual-Spatial Processing

**Mathematics Connection:** Students create and use visual representations to solve problems and to explore and communicate mathematical concepts and ideas in the mathematical domains. Visual-spatial processing affects students’ ability to solve multi-step problems, analyze certain representations, interpret and manipulate geometric configurations and appreciate changes in objects as they are moved in space.

**Student Learning Expectations:** Students should have opportunities to experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, and creating equations. Students will be expected to connect the different representations and explain the connections.

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<th><strong>Visual-Spatial Processing</strong></th>
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| Observations reveal difficulty perceiving numbers and performing operations with multiple digits, signs, and symbols. | The teacher will use and connect mathematical representations. (EMTP 3)  
- The teacher will provide tools, such as graph paper to align numbers, signs and symbols.  
- The teacher will utilize color coding to organize information to connect mathematical representations. | The student will use appropriate tools strategically. (MP.5)  
- The student will utilize technological tools to explore and deepen understanding of concepts. | **Graph Paper Math Intervention**  
This website describes how graph paper can be used as a math intervention. |
| Observations reveal difficulty identifying printed three-dimensional figures and other geometric objects. | The teacher will use and connect mathematical representations. (EMTP 3)  
- The teacher will provide concrete manipulatives to images to assist students in making mathematical connections. | The student will make connections between different mathematical representations and retain verbal description for objects and their individual characteristics. (MP.7 & 8) | **Teaching Geometrical Reasoning**  
This website contains activities for reinforcing Geometric Reasoning.  
**GeoModel Folding Shapes**  
This website provides access to concrete geometric three-dimensional figures. |
Descriptions

Behavior
Behavior is a form of communication. Every behavior has a function. A growth mindset in mathematics can be demonstrated through the students’ confidence and perseverance in their thinking and executing, asking questions for clarity, and grappling with context-based problems. These students are willing to tackle challenging problems and take learning risks.

Cognitive Processing
Cognitive processes allow students to make connections between prior concepts and new ideas. When students are aware and use at least one of the four mathematics cognitive processes, they will be capable of transferring cognitive skills into their learning.

Attention
Attention is a cognitive process that involves the student’s ability to listen to directions and explanations, participate in class discussions, focusing on different parts of the activity and stimuli at various moments in the lesson, and the ability to sustain an undisclosed amount of time dedicated to working and thinking on an activity for any given time range. Paying attention is learned.

Conceptual
Conceptual learning is when students use the previous learning experiences, ideas and concepts to understand and develop the new meaning of the new content by recognizing the connection and relationship of the old and new “building block concepts.”

Memory
Both long-term and short-term memory play essential roles in the learning of mathematics. Math is a conceptual subject consisting of connections and relationships.

Reasoning
Reasoning is the capacity to reflect on work, evaluate it, and then adapt, as needed.

Executive Functioning
Executive functioning is closely aligned to behavior and is judged by the strength of:
- self-awareness (self-directed attention),
- inhibition (self-restraint),
- non-verbal working memory (the ability to hold things in your mind),
- verbal working memory (self-speech),
- emotional self-regulation (learning to use words, images and your own self-awareness to process and alter how you feel about things),
- self-motivation (how well you motivate yourself to complete a task) and
- planning and problem solving (how you organize information in your brain).

Instructional Climate and Student Mindsets
Positive instructional climate and student mindsets can be developed through experiences obtained from physical location, ideas, and expectations of those within the environment. Classroom opportunities that demonstrate an awareness of and embrace all students’ ability to achieve success in mathematics can increase student autonomy and academic outcomes. Student autonomy encourages students to speak from their mathematical perspective and advocate for their own reasoning. Positive school climate leads to encouraging and supportive classroom environments that ultimately lead to growth mindsets and increased agency for each and every student.
**Language Processing**
Language processing involves students’ ability to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing. Language processing refers to the way humans use words to communicate ideas and understandings, and how such communications are processed and understood.

**Language Proficiency**
Mathematics language proficiency is measured in terms of interpretive and expressive language skills at the word/phrase/representation, sentence and discourse dimensions.

**Mathematics Calculation**
Mathematical calculations are basic computational skills (with and without the appropriate grade level calculator) and basic graphing skills that support the mathematical language, when solving contextual, mathematics problems. As a result of focusing on conceptual understanding, over time students develop procedural fluency (accurate, efficient, and flexible) and learn to make sense of and solve problems using automaticity of facts rather than rote recall.

**Other Exceptionalities**
Students with other exceptionalities may experience physical, social or emotional barriers that have the potential to influence how they engage in the learning experiences. Instructional opportunities should be limitless for all students to assist them with meeting high academic standards and developing mathematical reasoning and conceptual understanding of the content expectations.

**Problem-Solving**
Problem-solving is a cognitive process that involves students learning mathematics through contextual, mathematical problems and models. The contexts and models allow students to build meaning for the concepts so they can fully understand abstract concepts. Students are expected to use problem-solving strategies and develop these skills over time as they progress in the K - 12 curriculum.

**Visual-Spatial Processing**
Visual-spatial processing is a student’s ability to perceive, analyze, and understand visual information and communicate with the teacher and peers.
### English Language Proficiency for English (as a 2nd language) Learners

Students use their **emerging**, **developing**, or **expanding** 2nd language (English) to make meaning and engage with mathematics understandings. The student observations presented here will concentrate on students at the emerging, developing, and expanding levels of English language proficiency (ELP) as measured by the ACCESS for ELLs (CPL levels 2, 3, and 4) which describe most EL students. Recently arrived ELs at ELP 1 comprise <10% of the EL student population, and those who have achieved ELP 5 have exited the EL status and ESOL program services. It is expected that teacher’s will identify their student’s level of English proficiency to select the appropriate teacher actions.

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| Observations reveal **emerging or developing** understanding and interpretation of mathematical argumentation in English (**Interpretive Language Skills**: listening, viewing, and reading) | The teacher will support comprehension and make language demands of mathematics content visible by providing temporary scaffolds that include:  
• multimodal representations of vocabulary  
• multiple opportunities to make meaning using mathematical models, manipulatives, diagrams, drawings, graphs, and symbolic notations.  
• careful progression of mathematical activities and tasks  
• explicit teaching of English language for mathematical purposes  
• multiple opportunities for structed peer interactions to justify conclusions and critique others’ arguments using questions and negations. | The student will make sense of written or spoken mathematical conjectures, distinguish patterns among strategies used, and evaluate relationships between evidence and mathematical facts to create generalizations. | Language Demand in Mathematics Lessons (LDML) tool at www.nctm.org/more4u  
Integrating Language While Teaching Mathematics  
Integrating Language While Teaching Mathematics Teaching Practice Brief (ed.gov) |
| Observations reveal **emerging or developing** language to communicate mathematical arguments in English with precision and clarity. (**Expressive Language Skills**: Speaking and writing skills) | The teacher will support communication by providing temporary scaffolds that include:  
• teacher think-aloud & oral reasoning.  
• making sense of written instructions,  
• highlighting distinctions between meaning of terms used in | The student will record mathematical reasoning using precise and clear mathematical language in a variety of contexts to describe mathematical thinking to others.  
The student will develop English for mathematical purposes to present generalizable processes. | Language Demand in Mathematics Lessons (LDML) tool at www.nctm.org/more4u  
| Mathematics and terms used in everyday life, ● allowing processing time between first language and English, ● facilitating the use of tools (e.g., tables, graphs, cubes, calculators, electronic whiteboards, etc.) ● providing multiple opportunities for structured peer interactions. | **Mathematical Routines**
Mathematical Language Routines to Optimize Student Output |
| --- | --- |
| **Observations reveal expanding comprehension and interpretation of mathematical arguments in English.** *(Interpretive Language Skills: listening, viewing, and reading)* | The teacher will support comprehension by providing temporary scaffolds that include: ● making language visible through repeated teacher modeling and think-aloud, ● visual representation of word problems using charts, graphs, visual data, ● multiple opportunities for structured peer interactions or conversations | **Language Demand in Mathematics Lessons** *(LDML)* tool at [www.nctm.org/more4u](http://www.nctm.org/more4u)
| **Observations reveal expanding language to construct mathematical arguments in English.** *(Expressive Language Skills: Speaking and writingskills)* | The teacher will support students’ communication of mathematical explanations by providing temporary scaffolds that include: ● making English language visible through repeated teacher modeling of language connectors to order steps in a mathematical process *(first, next, then)* ● use of visuals (charts, graphs, diagrams, manipulatives, drawings) ● multiple opportunities for structured peer interactions or conversations | **Mathematical Language Routines** to Optimize Student Output ● Collect & Display ● Gather & Show Student Discourse ● Critique, Correct, and Clarify Scaffolds: ● Mathematics wordbanks ● Sentence frames ● Partner Talk ● Digital resources |
Additional Resources

Establish mathematics goals to focus learning.

- **10 Common Challenges Article**
  This article describes various ways teachers can make decisions based on their instructional goals.

- **Resources to Increase Students Interest**
  This is a website with different resources to increase student interest.

- **Standards Based Math Rubric**
  This resource is the learning progression for Math Standards.

Implement tasks that promote reasoning and problem solving.

- **GSE Effective Instructional Practices Guide**
  This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices.

- **Activities That Boost Spatial Reasoning**
  This website provides strategies to boost visual spatial reasoning.

- **Graph Paper Math Intervention**
  This website describes how graph paper can be used as a math intervention.

- **Step by Step Graphic Organizer**
  This resource is a Graphic Organizer to help students complete multi-step tasks/word problems.

- **Mathematics in the Real World**
  This website provides 22 examples of how mathematics is used in everyday life.

- **Multi-Sensory Instruction for Mathematics**
  10 ways to utilize multi-sensory instruction in the mathematics classroom.

Use and connect mathematical representations.

- **Information on KWL and KNWS Charts**
  This resource explains the use of KWL and KNWS charts in Mathematics.

- **The Positive Effect of Manipulatives in the Classroom**
  This website provides effective uses for concrete manipulatives.

- **Multi-Sensory Instruction for Mathematics**
  10 ways to utilize multi-sensory instruction in the mathematics classroom.

Facilitate meaningful mathematical discourse.

- **Mathematical Language Brainstorm**
  This resource is a student self-assessment/brainstorm graphic organizer.

- **Math Journal Examples**
  This resource provides many journal prompts and questions that elicit conceptual thinking and creativity.
Effective Word-Problem Instruction: Using Schemas to Facilitate Mathematical Reasoning

Academic Vocabulary Strategies
This website provides academic vocabulary strategies and resources to assist students with understanding mathematical language.

Multi-Sensory Instruction for Mathematics
10 ways to utilize multi-sensory instruction in the mathematics classroom.

Pose purposeful questions.

GSE Effective Instructional Practices Guide
This GA frameworks teacher guide to assist teachers in teaching students to think through the articulation and sharing of math strategies through 3 Act Tasks, Number Talks, and other formative instructional practices.

Is Your Answer Reasonable?
This lesson gives the teacher suggestions for guiding students in determining reasonableness.

Build procedural fluency from conceptual understanding.

Using Diagrams to Build and Extend Student Understanding
An example of how to model contextual situations to build and extend student understanding.

Multi-Sensory Instruction for Mathematics
10 ways to utilize multi-sensory instruction in the mathematics classroom.

Support efforts of learning in mathematics.

Chunking
This website explains chunking.

Three Reads Using a Problem Stem
Provides definition of Three Reads, background information, and sample Three Reads lessons.

Using Diagrams to Build and Extend Student Understanding
An example of how to model contextual situations to build and extend student understanding.

Elicit and use evidence of student thinking.

How to Use Choice Boards
This resource explains how to use choice boards in the classroom to differentiate assignments.

Student Rubric
This resource is a Self-Assessment Rubric.

Doodle Notes
This website provides Doodle Notes.

8 Strategies to Quickly Assess Prior Knowledge
Strategies for making purposeful connections in learning.
References


Celedón-Pattichis, S., & Ramírez, N.G. (2013) Advancing mathematics education for ELs – Beyond good teaching (NCTM)


